[**D**]

Sub-programs of the Main Program "LSSPHRD" for "*Light Scattering by a Spheroidal Particle*"

Sub-programs;

[ ] : Subroutine

{ } : External function

Subroutines

1) **ALEGQ** (X, M, NMAX, Q)

・Associated Legendre function of the second kind $Q_n^m(x)$ with real variable 'x'.
Arguments; Q (array) : $Q_0^m(x)$ to $Q_{\mathrm{NMAX}-1}^m(x)$.

2) **ANGLE** (ZETA, CHI, RLTHT, RLPHI, STHT, SPHI, SALPH)

・Transformation of the angular coordinates of an observing point from angles (Θ (RLTHT), Φ(RLPHI)) in the space system into angles (θ(STHT), ϕ(SPHI)) fixed to the spheroidal system for an orientation (ζ(ZETA), χ(CHI)) of the spheroid axis in the space.

3) **BOUNDR** (INCANG, M, KMIN, IMODE, RE1, RE2, RI1)

・Boundary conditions for determining the expansion coefficients $\alpha_{i,mn}$ and $\beta_{i,mn}$ of the scattered fields for the TE mode (i=1) and TM mode (i=2) incident waves, by solving the system of equations (86) (or Eqs.(127) and (128)) and (87) of AY1975 (Asano and Yamamoto, 1975).

・The determined coefficients are finally given in a complex 2-d array *ALBT*, which is transferred to COMMON /CMPLX/ *DI* in the main program.

Required subprogram; [SLEQDC]

4) **CDCOEF** (C, M, N, IPO, AINT, JWD, D, A, PI, SIGMA, DSUM, FCTR)

・Calculation of the eigenvalues λ_{mn} and the expansion coefficients d_r^{mn} of a spheroid with complex parameter 'c'.

Arguments ; Same as [RDCOEF], but for complex *'c'*.

Required subprograms; [SUBFF], {FF}, {FACTMM}

5) **CEIGEN** (A, R, N, MV)

・Calculation of eigenvalues and eigenvectors of a diagonal matrix with complex elements (copied from IBM Library)

Arguments; Same as [DEIGEN], but for complex elements.

6) **CEM** (M, N, C, XI, E, Y, AL, SUM, B, IN)

・Computation of the prolate radial functions of the second kind $R_{mn}^{(2)}(c;\xi)$ and their derivatives $\frac{dR_{mn}^{(2)}(c;\xi)}{d\xi}$,with complex parameter 'c' by the integral method by Sinha and MacPhie (1975).

Arguments; Same as [REM], but for complex 'c'.

Required subprogram; {DGAUSL}, {CFCT}

7) **CHODGE** (M, C, NX, Y, IPO)

・Estimation of initial values of λ_{mn} of a prolate spheroid with complex parameter 'c' by the matrix method of D. B. Hodge (1970).

Arguments; Same as [DMATR], but for complex 'c'.

Required subprogram; [DEIGQR]

8) **CLEGQ** (Z, M, NMAX, Q)

・Associated Legendre function of the second kind $Q_n^m(z)$ with complex variable 'z'.

Arguments; Q (array) : $Q_0^m(z)$ to $Q_{\mathrm{NMAX}-1}^m(z)$.

9) **CMATR** (M, C, NX, Y, IPO)

・Estimation of initial values of λ_{mn} of an oblate spheroid with complex parameter 'c' by the matrix method of D. B. Hodge (1970).

Arguments; Same as [DMATR], but for complex 'c'.

Required subprogram; [CEIGEN]

10) **COBSWF** (C, XI, M, N, MINT, INDX, NMAX, MAXJW, JW, D, ALAMD, SIGMA, DSUM, R1, DR1, R2, DR2)

・Calculation of oblate radial functions $R_{mn}^{(j)}(-ic;i\xi)$ and their derivatives $\frac{dR_{mn}^{(j)}(-ic;i\xi)}{d\xi}$, of the first ($j$ =1) and second (j=2) kinds with complex parameter 'c'.

Arguments; Same as [ROBSWF], but for complex parameter 'c'.

Required subprograms; [CLEGQ], [CMATR] , [CDCOEF]

11) **CPRSWF** (C, XI, M, N, MINT, INDX, NMAX, MAXJW, JW, D, ALAMD, SIGMA, DSUM, R1, DR1, R2, DR2)

・Calculation of prolate radial functions $R_{mn}^{(j)}(c;\xi)$ and their derivatives $\frac{dR_{mn}^{(j)}(c;\xi)}{d\xi}$, of the first ($j$ =1) and second (j=2) kinds with complex parameter 'c'.

Arguments; Same as [RPRSWF], but for complex 'c'.

Required subprograms; [ALEGQ], [CHODGE], [CDCOEF], [CEM]

12) **DEIGEN** (A, R, N, MV)

・Calculation of eigenvalues and eigenvectors of a diagonal, symmetric matrix with real elements (copied from IBM Library).

Arguments;

A (array): When called, the elements of the diagonal matrix.
When returned, the calculated eigenvalues at the diagonal positions in the decreasing order.

R (array): eigenvectors

N : dimension of the matrix

MV=0, when eigenvectors are needed.

MV=1, when eigenvectors are not needed.

13) **DEIGQR** (A,N,NA,H,NH,IPARAM,MULT,INT,LAMBDA,IER)

・Evaluation of eigenvalues and eigenfunctions for a non-Hermitian matrix with complex elements by the QR-method (copied from HITACHI Library).

14) **DMATR** (M, C, NX, Y, IPO)

・Estimation of eigenvalues λ_{mn} of a spheroid with real parameter 'c' by the matrix method of D. B. Hodge (1970).

Arguments;

Y (array) : calculated eigenvalues λ_{mn} to $\lambda_{m\,\mathrm{NX}}$.

IPO= 1 for prolate coordinate system

IPO= -1 for oblate coordinate system

Required subprogram; [DEIGEN]

15) **RDCOEF** (C, M, N, IPO, AINT, JWD, D, A, PI, SIGMA, DSUM, FCTR)

・Calculation of the eigenvalues λ_{mn} and the expansion coefficients d_r^{mn} of a spheroid with real parameter $c = 2\pi l/\lambda$.

Arguments;

IPO= 1 for prolate coordinate system; IPO= -1 for oblate coordinate system.

D (array) : expansion coefficients d_r^{mn} $(r = 0, \cdots, JWD)$.

AINT : initial value of the eigenvalue.

A : corrected eigenvalue λ_{mn} .

PI = $\sum' \frac{(2m+r)!}{r!} d_r^{mn}$.

DSUM : summation of the expansion coefficients ($\sum' \; d_r^{mn}$).

SIGMA : normalization constant Λ_{mn} of the expansion coefficients (Eq.(18) in AY1975).

FCTR : index for computational overflow check.

Required subprograms; [SUBFF], {FF}, {FACTMM}

16) **REM** (M, N, C, XI, E, Y, AL, SUM, B, IN)

・Computation of the prolate radial functions of the second kind $R_{mn}^{(2)}(c;\xi)$ and their derivatives $\frac{dR_{mn}^{(2)}(c;\xi)}{d\xi}$, with real parameter '$c$' by the integral method of Sinha and MacPhie (1975).

Arguments;

E : eigenvalue $\lambda_{mn}(c)$.

Y = $\int_0^\infty \mathrm{e}^{-y}\,\{\mathrm{RFCT}\}dy$.

AL = $\alpha \equiv \log(XI)$; B = β; SUM = $\sum_{k=1}^{5} \frac{\delta_k}{(m+N)^k}$

IN = 0 for $R_{mn}^{(2)}(c;\xi)$; IN = 1 for $\frac{dR_{mn}^{(2)}(c;\xi)}{d\xi}$

Required subprogram; {DGAUSL}, {RFCT}

17) **ROBSWF** (C, XI, M, N, MINT, INDX, NMAX, MAXJW, JW, D, ALAMD, SIGMA, DSUM, R1, DR1, R2, DR2)

・Computation of oblate radial functions $R_{mn}^{(j)}(-ic;i\xi)$ and their derivatives $\frac{dR_{mn}^{(j)}(-ic;i\xi)}{d\xi}$, of the first ($j$ =1) and second (j=2) kinds with real parameter 'c'

Arguments ; Same as [RPRSWF], but for oblate system.

Required subprograms: [CLEGQ], [DMATR] , [RDCOEF]

18) **RPRSWF** (C, XI, M, N, MINT, INDX, NMAX, MAXJW, JW, D, ALAMD, SIGMA, DSUM, R1, DR1, R2, DR2)

・Calculation of prolate radial functions $R_{mn}^{(j)}(c;\xi)$ and their derivatives $\frac{dR_{mn}^{(j)}(c;\xi)}{d\xi}$, of the first ($j$ =1) and second (j=2) kinds with real parameter 'c'.

Arguments;

MINT : Index parameter of calculation of Bessel functions (calculated for m=MINT).

NMAX: Number of eigenvalues ($\lambda_{m,m}$~$\lambda_{m,\mathrm{NMAX}-1}$) estimated by the subprogram [DMATR].

MAXJW: Maximum number of expansion coefficients $d_r^{mn}(c)$.

INDX=1 for the external field ; INDEX=2 for the internal field.

D (array) : expansion coefficients d_r^{mn} .

ALAMD : eigenvalue λ_{mn}.

SIGMA : normalization constant Λ_{mn} of the expansion coefficients (Eq.(18) in AY1975).

DSUM : summation of the expansion coefficients ($\sum' \ d_r^{mn}$).

Required subprograms; [ALEGQ], [DMATR], [RDCOEF], [REM]

19) **SLEQDC** (N, NSYS)

・Solutions of multi-system of complex linear equations, $[AL] \times [X] = [AR]$, by means of the elimination method. The elements of $[AL]$ and $[AR]$ are transferred via COMMON /SLEQ/ between [BOUNDR]. The solutions X are given in the array $[AR]$ in the original order.

Arguments;

N : dimension of the matrix

NSYS : number of systems

20) **SUBFF** (M, N, FF, COEF)

・ Subroutine version of the external function {FF} to calculate factorial ratios.

Arguments;

FF : computed value of factorial term, see {FF(M, N)}.

COEF : coefficient for computational overflows.

21) **UVXYAI** (C, C2, XI, M, JEM, JIN, NN, LL, UE1, UE3, UI1, VE1, VE3, VI1, XE1, XE3, XI1, YE1, YE3, YI1, IOP)

・ Calculations of the parameters, $U_{mn}^{(j),t}$, $V_{mn}^{(j),t}$ $X_{mn}^{(j),t}$ and $Y_{mn}^{(j),t}$, defined by Eqs. (75) to (78) of AY1975 in the prolate system and Eqs. (79) to (82) in the oblate system, and the expansion coefficients, A_t^{mn} through I_t^{mn}, defined by Eqs.(60) through (73), for the external (E) and internal (I) fields of the spheroid.

・COMMON /XYUV/ SPCI, RI1, DRI1, RE3, DRE3, DE, RE1, DRE1, SPCE
COMMON /CMPLX/ DI

Arguments;

IPO= 1 for prolate coordinate system,

IPO= -1 for oblate coordinate system.

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22) SUBROUTINE **ROTAT** (ANGLE)

・Transformation matrix $\boldsymbol{L}(-\alpha)$ of the Stokes parameters (I_l, I_r, U, V) for a rotation of the reference plane by an angle α.

・Called from the main program "**LSRND3D"** to compute "*Light Scattering by Randomly Oriented Spheroidal Particles*" by Asano and Sato (1980).

External Functions

23) **FACTMM** (M)

・Calculation of factorial; FACTMM (m) = (m+m)!/m!

24) **FF** (M, N)

・Calculation of the following factorial terms;

$$\mathrm{FF(m,n)} = \frac{2^{\mathrm{n-m}}\left(\frac{\mathrm{n-m}}{2}\right)!\left(\frac{\mathrm{n+m}}{2}\right)!}{(\mathrm{n+m})!}, \quad (\mathrm{n-m}) = \mathrm{even},$$
$$\frac{2^{\mathrm{n-m}}\left(\frac{\mathrm{n-m-1}}{2}\right)!\left(\frac{\mathrm{n+m+1}}{2}\right)!}{(\mathrm{n+m+1})!}, \quad (\mathrm{n-m}) = \mathrm{odd}.$$

25) **CFCT** (X)

・Integrant function in the subroutine [CEM] for the integral method by Sinha and MacPhie (1975) to compute $R_{mn}^{(2)}(c;\xi)$ for the case of complex 'c'. Same as {RFCT}, but for complex 'c'.
Parameters are transferred via COMMON /R2INTC/ between [CEM].

26) **DGAUSL** (FCT, N)

・Gaussian-Laguerre quadrature of an integral, $\int_0^\infty \mathrm{e}^{-x} f(x)dx$, with 4*n* points ($n = 1, \cdots, 6$) (copied from HITACHI Library).

Arguments; FCT = $f(x)$; N = *n*.

27) **RFCT** (X)

・Integrant function in the subroutine [REM] for the integral method by Sinha and MacPhie (1975) to compute $R_{mn}^{(2)}(c;\xi)$ for the case of real 'c'. The parameters are transferred via COMMON /R2INTR/ between [REM].

$$\mathrm{RFCT}(x) = (\frac{x}{\alpha} + N + m + 2)^{\beta} \cdot \exp\left[\sum_{k=1}^{5} \frac{\delta_k}{(x/\alpha + N + m + 2)^k}\right].$$

References

Asano, S. and G. Yamamoto, 1975: Light scattering by a spheroidal particle, *Appl. Optics*, **14**, 29-49.

Asano, S., 1979: Light scattering properties of spheroidal particles, *Appl. Optics*, **18**, 712-723.

Asano, S. and M. Sato, 1980: Light scattering by randomly oriented spheroidal particle, *Appl.Optics*, **19**, 962-974.

(S. Aasano)