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SUBROUTINES FOR COMPUTING THE PARAMETERS OF THE ELECTROMAGNETIC RADIATION SCATTERED BY A SPHERE

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1. INTRODUCTION

The passage of the electromagnetic radiation through a medium is generally accompanied by the removal of a fraction of the energy from the incident beam. This fraction may get partly absorbed within the medium, and may get partly scattered, i.e., reappear in same as well as other directions with or without change in frequency. Even when the scattering process does not involve any change in frequency (a case to which this report is restricted), the characteristics of the scattered radiation are determined by the wavelength (λ) of the incident radiation, refractive index ($m = n_1 - i\,n_2$) of the medium, and size as well as the shape of the discrete particles in the medium. Because of this, measurements and proper interpretation of the characteristics of the scattered radiation offer a very good opportunity for obtaining information about the state of the medium. Hence, numerical determination of the characteristics of the scattered radiation for a given model is of prime importance in several diversified fields such as planetary and atmospheric optics, astrophysics, laser, radar and searchlight applications, and chemistry. To this end, the first step is the evaluation of the characteristics of the radiation scattered by a single particle.

The numerical evaluation of the parameters of the radiation scattered by a particle very small compared to λ is very simple and straightforward (Rayleigh, 1871). The expressions for the radiation scattered by a sphere whose radius (r) is comparable to, or greater than, λ were first derived by Mie (1908) and as such,

this process of scattering is known as Mie scattering. The expressions for Mie scattering involve series whose terms contain Bessel functions of half-integral order (spherical Bessel functions) with complex argument, and, first and second derivatives of the Legendre polynomials. Furthermore, the number of terms required for evaluating the series with reasonable accuracy is of the order of the size parameter $x = 2\pi r/\lambda$. Hence, the reliable Mie scattering computations for large spheres can be described as difficult, tedious and time consuming.

The derivations of the Mie expressions as well as a catalog of its available numerical solutions in some specific cases can be found in the treatise, "Light Scattering by Small Particles" by Van de Hulst (1957). Prior to this publication, numerical evaluation of the Mie expressions was confined to small values of x (≤ 10) for partly-absorbing ($n_2 > 0$) spheres. For non-absorbing spheres ($n_2 = 0$), the most outstanding work is that of Gumprecht and Sliepcevich (1951, 1953), who carried their calculations to $x = 400$. As has been pointed out by Van de Hulst, the work of these authors cannot be considered to be complete or the laws of geometric and physical optics can be applied to the transfer problem in a sphere with some reasonable confidence only if x is of the order of 2000. Furthermore, the number of directions for which the radiation parameters are given by these authors is too small to give any insight into the fine structure of the radiation field. Besides, many fields of application demand extensive numerical data for values of refractive indices other than those used by Gumprecht and Sliepcevich.

Since 1957, some notable progress has been made in extending the range of x for partly-absorbing spheres (e.g., Giese, 1961 ; Deirmendjian, Clasen and Viezee, 1961 , Irvine, 1963, 1965 ; Cunningham, Wells and Collins, 1966 ; Plass, 1966 ; and Kattawar and Plass, 1967) . However, surprisingly enough, no results have been reported for large spheres ($x > 400$) which are partly-absorbing or non-absorbing (see Footnote on p. 40).

This absence of progress may be attributed partly to the limited interest of various workers. However, since Mie computations involve evaluation of series whose terms increase by several orders of magnitude with increase of n and change sign in rather uneven manner, it is essential to carry out basic arithmetic in double precision if x is large. The other factor is the large storage requirements. Computations of the radiation scattered in 37 different directions [0(5) 180] by a sphere having size parameter $x = 1000$ with straighforward programming procedure can require use of more than one hundred thousand double precision words of storage, a requirement too large to be fulfilled by the main storage area of any modern computer. Hence, one is required to use magnetic tapes or disks.

The main difficulty in evaluation of the characteristics of the radiation scattered by a large, partly-absorbing sphere is the very rapid propagation of the errors when the function $A_n(mx)$ is computed using a recurrence relationship (sec. 3.2). A recurrence relationship provides a very powerful computing tool, especially in automatic work. However, since generation is carried out perforce

with rounded values, the errors may or may not grow relative to the size of the wanted function. If the errors do grow, the recurrence scheme is said to be unstable. When the upward recurrence scheme (i.e., starting with the value of $A_0(mx)$, one computes successively higher values of $A_n(mx)$ by making use of recurrence formula) is unstable, in general the downward recurrence scheme is found to be very stable (Abramowitz and Stegun, 1964, p. XIII). It is not necessary to know the initial value for starting the downward recurrence scheme; one can make use of the ratio method first described by Miller (1952). Reference may also be made of an independent later work in this direction by Corbato and Uretsky (1959), who have applied the downward recurrence procedure for generation of spherical Bessel functions on digital computers. Kattawar and Plass (1967) seem to be the first to use this downward recurrence procedure in Mie scattering calculations.

The purpose of this report is to describe and to present two FORTRAN subroutines with the help of which one can evaluate the characteristics of the electromagnetic radiation scattered by a sphere, in as many as 200 different directions. The subroutines are written using FORTRAN IV H language and IBM System/360 computers in mind.

In the first subroutine called DAMIE, all the functions are computed using the upward recurrence procedure and as such the storage requirement is very modest (15474 bytes for any value of the size parameter). However, because of

the use of upward recurrence, the scheme becomes unstable for $A_n(mx)$ when $n_2 > 0$, i.e. the spheres are partly-absorbing. From the discussion in sec. 3.2, it will be clear that further work is needed in understanding the problem of the error propagation in the case of function $A_n(mx)$. The only positive statement which can be made at this stage, is as follows. The final results obtained using DAMIE should not be accepted without checking them against those obtained using DBMIE if the parameter $n_2 x$ is about 50 or higher, or x is of the order of 100 or more.

In the second subroutine called DBMIE, the function $A_n(mx)$ is computed using downward recurrence procedure and hence, all its values have to be stored for further use. This does not lead to any significant increase in computer time but the storage requirement is very high. Since the recurrence is started with

$$A_N(mx) = 0.0 + 0.0\,i \qquad (1)$$

where

$$N = 1.1\,(n_1^2 + n_2^2)^{1/2}\,x + 1 \quad , \qquad (2)$$

a storage of 127340 bytes can accomodate x of about 20 if $n_1 = n_2 = 200$, a case for which some results have been presented by Van de Hulst (1957, p. 293). However, the results obtained using DBMIE are expected to be more reliable than those obtained with DAMIE under all foreseeable conditions.

II. THEORY

2.1 Stokes Parameters:

A beam of electromagnetic radiation is vectorial in nature as it requires more than one (actually four) parameters to fully describe its state. These parameters, as customarily understood, are as follows:

1. Specific Intensity (I): It is the amount of energy flowing normally through unit area in a cone of unit solid angle per unit time and per unit frequency interval. It is also referred to as Intensity, Radiance or Brightness.

2. Degree of Polarization (P): As a Nicol prism (analyser) is rotated around the axis of observation, the intensity of the source as seen through the prism, undergoes variations especially if the source consists of some scattered light, or alternately, if the beam happens to pass through some polarizing medium before reaching the Nicol prism ** this occurs, the intensity passes through a maximum value (I_{max}) and a minimum value (I_{min}) twice during a 360° rotation. The positions of I_{max} and I_{min} are at right angles to each other. The degree of polarization of the observed radiation is given by

$$P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \quad . \qquad (3)$$

3. Deviation of Plane of Polarization (X): For several reasons, it may be convenient to refer to the directions of I_{max} and that of I_{min} with respect to two arbitrary directions (say e and r) which are mutually at right angles to each other. The e-r plane is assumed to be at right angles to the direction of observation. If the direction of polarization, i.e., that of I_{max}, makes an angle X, say with that of e, then the plane of polarization is said to deviate by an angle X.

4. Ellipticity of Polarization (Δ): The phenomenon of polarization is best understood by imagining two vibrations of complex amplitudes E_{max} and E_{min} along the direction of I_{max} and I_{min} respectively. Then, $I_{max} = |E_{max}|^2$ and $I_{min} = |E_{min}|^2$ if the constant of proportionality is assumed to be unity. If these vibrations maintain a constant phase difference (Δ) between them over millions of vibrations, the radiation is said to be elliptically polarized. [This restriction (millions of vibrations) is necessary because one vibration takes of the order of 10^{-8} seconds and any available instrument averages over a period of the order of a second.] The detection of this ellipticity requires the use of a wave plate (an optical component which introduces some arbitrary phase difference between the two vibrations) and also the Nicol prism.

This representation of the state of a beam of electromagnetic radiation by the customary parameters (I, P, X and Δ) though complete and easily understood, leads

to unwanted complications when one is interested, e.g., in obtaining the results of interaction of several beams on the same particle. This is because these customary parameters are of different dimensions and are non-additive. This difficulty can be bypassed by making use of the following alternate set of parameters first proposed by Stokes (1852):

In the Stokes representation, the intensity $\underline{I}$ is treated as a one-column matrix of four elements (or alternately, a vector with four elements). All the elements have the same dimensions and are also additive.

$$\underline{I} = \begin{vmatrix} I \\ Q \\ U \\ V \end{vmatrix} \tag{4}$$

For the definition of these elements, let us assume two directions, e and r, at right angles to each other, such that the er plane is perpendicular to the direction of the propagation of the radiation under study. Let us further assume that the complex amplitudes of electrical vibrations are E_e and E_r, respectively, along the e- and r- directions. Their complex conjugates will be denoted by E_e^* and E_r^*. Again assuming the constant of proportionality to be unity, the four elements are defined as follows:

$$\left.\begin{aligned} I &= E_e E_e^* + E_r E_r^* , \\ Q &= E_e E_e^* - E_r E_r^* , \\ U &= E_e E_r^* + E_r E_e^* \end{aligned}\right\} \tag{5}$$

and $$V = i(E_e E_r^* - E_r E_e^*) .$$

This matrix, though symmetric, is less simple than the following somewhat less symmetric matrix which we shall also refer to as the Stokes matrix:

$$\underline{I} = \begin{vmatrix} I_e \\ I_r \\ I_u \\ I_v \end{vmatrix} \tag{6}$$

where the elements of the two matrices are related as follows:

$$\left.\begin{aligned} I &= I_e + I_r , \\ Q &= I_e - I_r , \\ U &= I_u , \end{aligned}\right\} \tag{7}$$

and $$V = I_v .$$

The following relationships can be shown to exist between the Stokes and the customary parameters of the beam of electromagnetic radiation (e.g., Chandrasekhar, 1950 ; Van de Hulst, 1957) :

$$P = \frac{\left[(I_e - I_r)^2 + I_u^2 + I_v^2\right]^{1/2}}{I_e + I_r} , \qquad (8)$$

and

$$\tan 2X = \frac{I_u}{I_e - I_r} \qquad (9)$$

In order to relate δ to the Stokes parameters, we have to define an angle β given by

$$\sin 2\beta = \frac{I_v}{\sqrt{(I_e - I_r)^2 + I_u^2 + I_v^2}} , \qquad (10)$$

then

$$\tan \delta = \frac{\tan 2\beta}{\sin 2X} \qquad (11)$$

If there is no elliptical polarization, i.e., $\delta = 0$, the parameter $I_v = 0$. A partially polarized beam of radiation can then be represented by three parameters, I_e, I_r, and I_u. If the direction of maximum should coincide with that of e or r, i.e., $X = 0^\circ$ or 90°, then $I_u = 0$ and

$$P = \frac{I_r - I_e}{I_r + I_e} . \qquad (12)$$

A natural beam of radiation which is completely unpolarized has for its Stokes parameters I/2, I/2, 0, 0.

2.2 Expressions for Mie Scattering

The expressions for the radiation scattered by a sphere of radius (r), and of material with a complex index of refraction (m), have been aptly derived by Van de Hulst (1957), and also by Born and Wolf (1964). We shall therefore enumerate the final results only.

Let $\underset{\sim}{I}_i$ and $\underset{\sim}{I}_s$ respectively represent the Stokes parameters of the radiation incident on and scattered by a sphere. Then,

$$\underset{\sim}{I}_s = \underset{\sim}{F}' \cdot \underset{\sim}{I}_i \qquad (13)$$

where $\underset{\sim}{F}'$ is a four-by-four matrix referred to as a "transformation matrix" by Van de Hulst (1957, p. 44). It has the following form

$$\underset{\sim}{F}' = \begin{vmatrix} M_2 & 0 & 0 & 0 \\ 0 & M_1 & 0 & 0 \\ 0 & 0 & S_{21} & -D_{21} \\ 0 & 0 & D_{21} & S_{21} \end{vmatrix} . \qquad (14)$$

The matrix $\underline{F}'$ and hence the matrix $\underline{L}_s$ are the functions of the following parameters:

$x = 2\pi r/\lambda$, where λ is the wavelength of the incident radiation;

$m = n_1 - in_2$, index of refraction of the material of the sphere with respect to its surroundings;

and θ = the angle between the direction of the incident and the scattered radiation.

Van de Hulst has also noted that only three elements of $\underline{F}'$ are independent, the interrelationship being

$$S_{21}^2 + D_{21}^2 = M_2 M_1 \quad . \tag{15}$$

However, as S_{21} and D_{21} can be positive or negative, no useful purpose is served by using this relationship in actual computations.

In order to evaluate these elements, one first defines the complex amplitudes $S_1(x, m, \theta)$ and $S_2(x, m, \theta)$ for the scattered radiation.

$$S_1(x, m, \theta) = \sum_{n=1}^{\infty} \frac{(2n+1)}{n(n+1)} \left\{ a_n(x, m)\, \pi_n(\mu) + b_n(x, m)\, \tau_n(\mu) \right\} \tag{16}$$

and

$$S_2(x, m, \theta) = \sum_{n=1}^{\infty} \frac{(2n+1)}{n(n+1)} \left\{ b_n(x, m)\, \pi_n(\mu) + a_n(x, m)\, \tau_n(\mu) \right\} \tag{17}$$

where

$$\mu = \cos\theta \quad . \tag{18}$$

Then,

$$\begin{aligned} M_1 &= S_1 S_1^* \quad , \\ M_2 &= S_2 S_2^* \quad , \\ S_{21} &= \tfrac{1}{2}\left(S_2 S_1^* + S_1 S_2^*\right) \quad , \\ \text{and} \quad D_{21} &= \frac{i}{2}\left(S_2 S_1^* - S_1 S_2^*\right) \quad . \end{aligned} \tag{19}$$

The functions $a_n(x,m)$ and $b_n(x,m)$ are given by

$$a_n(x,m) = \frac{\Psi_n'(mx)\,\Psi_n(x) - m\,\Psi_n(mx)\,\Psi_n'(x)}{\Psi_n'(mx)\,\xi_n(x) - m\,\Psi_n(mx)\,\xi_n'(x)} \quad , \tag{20}$$

and

$$b_n(x,m) = \frac{m\,\Psi_n'(mx)\,\Psi_n(x) - \Psi_n(mx)\,\Psi_n'(x)}{m\,\Psi_n'(mx)\,\xi_n(x) - \Psi_n(mx)\,\xi_n'(x)} \tag{21}$$

$\Psi_n'(z)$ and $\xi_n'(x)$ are the respective derivatives of $\Psi_n(z)$ and $\xi_n(x)$ with respect to z and x respectively.

$$\Psi_n(z) = z\, j_n(z) \tag{22}$$

$$\xi_n(x) = x\,[\, j_n(x) - i\, y_n(x)\,] \tag{23}$$

$$\Psi_n'(z) = z\, j_{n-1}(z) - n\, j_n(z) \tag{24}$$

and $$\xi_n'(x) = x\,[\, j_{n-1}(x) - i\, y_{n-1}(x)\,] - n\,[\, j_n(x) - i\, y_n(x)\,] \tag{25}$$

j_n and y_n are the spherical Bessel functions of the first and second kind, respectively. The following recurrence relationships and initial values for setting up the recurrence procedure can be found in any of the standard mathematical books (e.g., Antosiewicz, 1964) .

$$f_{n+1}(z) = \frac{2n+1}{z}\, f_n(z) - f_{n-1}(z) \tag{26}$$

$$[\, f_n(z) : j_n(z),\ y_n(z)\,] \ .$$

$$j_{-1}(z) = -y_0(z) = \frac{\cos z}{z} \ , \tag{27}$$

and

$$j_0(z) = y_{-1}(z) = \frac{\sin z}{z} \ . \tag{28}$$

The phase functions $\pi_n(\mu)$ and $\tau_n(\mu)$ appearing in eqs. (16) and (17) can be expressed in terms of the Legendre polynomials as follows:

$$\pi_n(\mu) = \frac{d\, P_n(\mu)}{d\mu} \ , \tag{29}$$

and

$$\tau_n(\mu) = \mu\, \pi_n(\mu) - (1-\mu^2)\, \frac{d\, \pi_n(\mu)}{d\mu} \ . \tag{30}$$

Besides the transformation matrix $\underline{F}$, other terms of considerable importance are the dimensionless constants referred to as "efficiency factors" by Van de Hulst (1957, p. 14 and p. 127). Q_e, the efficiency factor for extinction, which is the ratio of the total amount of energy removed from the incident beam to the geometric cross-section (πr^2) of the particle, can be obtained from the values of a_n and b_n only.

$$Q_e(x,m) = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1)\, [\, \mathrm{Re}(a_n) + \mathrm{Re}(b_n)\,] \ . \tag{31}$$

The symbol Re stands for the real part of the quantity in parentheses.

Q_s, the efficiency factor for scattering, is given by the following expression:

$$Q_s(x,m) = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1)\, [\, |a_n|^2 + |b_n|^2\,] \tag{32}$$

If there is no absorption, i.e., $n_2 = 0$, $Q_e = Q_s$. Otherwise, Q_a, the efficiency factor for absorption, is given by

$$Q_a = Q_e - Q_s . \tag{33}$$

Another dimensionless quantity of considerable interest is the so-called "asymmetry factor" represented by $\overline{\cos\theta}$.

$$\overline{\cos\theta} = \frac{4}{x^2 Q} \sum_{n=1}^{n} \left\{ \frac{n(n+2)}{n+1} \operatorname{Re}\left(a_n a_{n+1}^{*} + b_n b_{n+1}^{*}\right) + \frac{2n+1}{n(n+1)} \operatorname{Re}\left(a_n b_n^{*}\right) \right\} \tag{34}$$

As noted by Irvine (1963), and again by Kattawar and Plass (1967) [but not by Giese (1961)], asterisks appearing in eq. (34) have been omitted in Van de Hulst (1957).

Q_p, the efficiency factor for radiation pressure, can then be obtained using the following:

$$Q_p = Q_e - \overline{\cos\theta} \cdot Q_s \tag{35}$$

III. A SECOND LOOK AT THE MIE EXPRESSIONS

Since the evaluation of the MIE amplitudes S_1 and S_2 can lead to the computations of a thousand or more terms by recurrence, it is advisable to give some thought to the question of the propagation of errors in the case of each of the four functions.

3.1 a_n and b_n:

The expressions for the functions a_n and b_n (eqs. (20) and (21)) for complex values of the parameter m contain the spherical Bessel functions $j_n(mx)$ which in turn require hyperbolic sine and hyperbolic cosine functions $\sinh(n_2 x)$ and $\cosh(n_2 x)$. As is well-known, the values of these hyperbolic functions increase exponentially with $n_2 x$. Since the computers deal only with finite numbers, the evaluation of the expressions in the present form can result in an overflow if $n_2 x \sim 170$ and IBM System/360 computers are used.

This overflow can be avoided by dividing both numerator and denominator of the expression for a_n as well as that of b_n by $j_n(mx)$ as all the terms contain $j_n(mx)$ or $j_{n-1}(mx)$. Infeld (1947) who seems to be the first to recognize this, introduced the so-called logarithmic derivative of $\psi_n(mx)$ and $\xi_n(x)$ functions given by eqs. (22) and (23), respectively. Some of the properties of these logarithmic derivative functions have been discussed by Aden (1951). The function

$$\frac{d[\log \Psi_n(mx)]}{d(mx)} = \frac{\Psi_n'(mx)}{\Psi_n(mx)} \qquad (36)$$

is denoted as $a_n(mx)$ by Aden (1951), as $A_n(mx)$ by Deirmendjian and Clasen (1962), and as $D_n(mx)$ function by Kattawar and Plass (1967). Using the notations of Deirmendjian and Clasen (1962), we have,

$$a_n(x,m) = \frac{\left\{\frac{A_n'}{m} + \frac{n}{x}\right\} Re[W_n(x)] - Re[W_{n-1}(x)]}{\left\{\frac{A_n(mx)}{m} + \frac{n}{x}\right\} W_n(x) - W_{n-1}(x)}, \qquad (37)$$

and

$$b_n(x,m) = \frac{\left\{m A_n(mx) + \frac{n}{x}\right\} Re[W_n(x)] - Re[W_{n-1}(x)]}{\left\{m A_n(mx) + \frac{n}{x}\right\} W_n(x) - W_{n-1}(x)}. \qquad (38)$$

The logarithmic derivative of $\Psi_n(mx)$, i.e., the function $A_n(mx)$, has the following form:

$$A_n(mx) = -\frac{n}{mx} + \frac{J_{n-1}(mx)}{J_n(mx)} = -\frac{n}{mx} + \frac{1}{\frac{n}{mx} - A_{n-1}(mx)}. \qquad (39)$$

The following initial value for setting up the recurrence can be obtained after making use of eqs. (27) and (28):

TABLE 1

Notations as used by Deirmendjian and Clasen (1962), and by Kattawar and Plass (1967).

Deirmendjian and Clasen	Kattawar and Plass
$Re[W_n(x)]$	$Y_n(x)$
$W_n(x)$	$\xi_n(x)$
$A_n(mx)$	$D_n(mx)$
$\frac{Re[W_{n-1}(x)]}{Re[W_n(x)]} - \frac{n}{x}$	$D_n(x)$
$\frac{W_{n-1}(x)}{W_n(x)} - \frac{n}{x}$	$G_n(x)$

$$A_o(mx) = \frac{J_{-1}(mx)}{J_0(mx)} = \cot(mx) \quad . \tag{40}$$

The function $W_n(x)$ appearing in eqs. (37) and (38) is the same as $\xi_n(x)$ defined by eq. (22). Hence, after making use of eqs. (26) to (28), we have

$$W_n(x) = \frac{2n-1}{x} W_{n-1}(x) - W_{n-2}(x) \quad , \tag{41}$$

with

$$W_{-1}(x) = \cos x - i \sin x \quad , \tag{42}$$

and

$$W_0(x) = \sin x + i \cos x \quad . \tag{43}$$

At the first sight, modified expressions of $a_n(x,m)$ and $b_n(x,m)$ as given by Kattawar and Plass (1967) appear to be different from those given by Deirmendjian and Clasen (1962). This is not really the case and one can check their equivalence by making use of Table 1.

3.2 A_n and W_n:

The initial value for setting up the recurrence procedure for computations of $A_n(mx)$ is given by eq. (40). With $m = n_1 - in_2$, it can be written in either of the following forms:

$$A_o(mx) = \frac{\sin(2n_1x) + i \sinh(2n_2x)}{\cosh(2n_2x) - \cos(2n_1x)} \quad , \tag{44}$$

or

$$A_o(mx) = \frac{\sin(n_1x)\cos(n_1x) + i \sinh(n_2x)\cosh(n_2x)}{\sin^2(n_1x) + \sinh^2(n_2x)} \quad . \tag{45}$$

If $n_2x = 0$ and n_1x is an integer multiple of π, either of these forms will give rise to a division by zero and hence, a possible termination of the program, or wrong results. This is not a hypothetical case, as several situations can be visualized where $2n_1\pi/\lambda$ can be an integer. Because of the generation of the round-off errors in a particular way, the expression for $A_o(mx)$ given by eq. (45) was found to be more suitable for computational purposes. However, it will be necessary to apply due caution if $2n_1\pi/\lambda$ is very close to an integer, since round-off errors are generated differently in different computers.

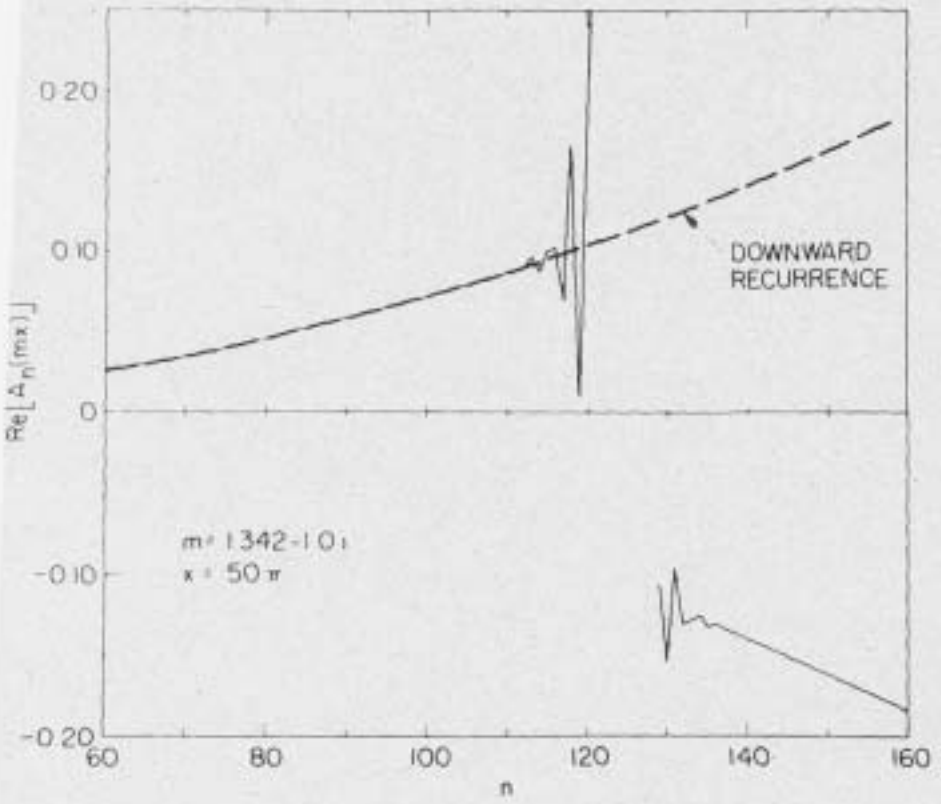
If $n_2 \neq 0$, a straightforward evaluation of $A_o(mx)$ will again run into trouble $n_2x \sim -90$ or more. Since for these large values of n_2x, $\sinh(n_2x) \sim \cosh(n_2x) \sim \frac{1}{2} e^{(n_2x)}$, we may make use of the following approximation:

$$A_o(mx) = 0.0 + 1.0\,i \tag{46}$$

A use of this approximation immediately leads to the question of propagation of errors, as the successive values of $A_n(mx)$ are computed by upward recurrence

FIGURE 1

Variation of real part of $A_n(mx)$ as a function of n, $m = 1.342 - 1.0\,i$, $x = 50.0\,\pi$. Broken curve represents values obtained using downward recurrence procedure. Thin solid curve represents values obtained using upward recurrence procedure.



[eq. (39)]. As mentioned earlier (sec. 1), Kattawar and Plass (1967) have already pointed out that a numerical instability develops in this case. For setting up the downward recurrence procedure, they have suggested the use of $0.0 + 0.0\,i$ as the initial value of $A_N(mx)$ for some value of $N > |mx|$. Since the computational error decreases very rapidly at each step of downward recurrence in this particular case, the calculations are insensitive to the assumed starting value and converge very rapidly to the correct value. The criterion for selecting downward recurrence at a value of N given by eq. (2) is rather strict.

The real part of the function $A_n(mx)$ for $m = 1.342 - 1.0\,i$ and $x = 50.0\,\pi$ as obtained after using the upward recurrence (thin solid curve) and downward recurrence (broken curve) procedures, is plotted in Figure 1 as a function of n. It can be seen that a numerical instability develops in the upward recurrence scheme when $n = 111$. Between 112 and 125, large oscillations develop. The amplitude of these oscillations decreases when n is about 130. For $n = 136$ and higher, a stabilization occurs, but the real part of $A_n(mx)$ achieves a value which is completely different from the correct value given by the broken curve.

The ultimate effect of this instability is to increase the values of the efficiency factor Q_s [eq. (32)]. Eventually, Q_s becomes greater than Q_e leading to negative values for Q_a [eq. (33)]. An example of this is provided in Table 2 where the values of Q_n as obtained using the upward recurrence procedu

TABLE 2

The values of the efficiency factor for absorption (Q_a) as a function of n_2 : $m = 1.342 - n_2 i$, $r = 10\mu$, $\lambda = 0.4\mu$, $x = 50.0\pi$.

Column 2: Values obtained when function $A_n(mx)$ is computed using upward recurrence procedure (DAMIE).

Column 3: Values obtained when function $A_n(mx)$ is computed using downward recurrence procedure (DBMIE).

n_2	Q_a with DAMIE	Q_a with DBMIE
0.0	0.0000	0.0000
0.0001	0.0535	0.0535
0.001	0.4149	0.4149
0.01	0.9649	0.9649
0.1	0.9653	0.9653
0.2	0.9542	0.9542
0.3	0.9389	0.9390
0.4	0.4913	0.9211
0.5	- 0.4925	0.9016
0.6	- 0.5265	0.8808
0.7	- 1.1592	0.8592
0.8	- 2.3079	0.8369
0.9	- 1.4379	0.8141
1.0	- 1.3789	0.7910

(subroutine DAMIE), and as obtained using the downward recurrence procedure (subroutine DBMIE) for computations of $A_n(mx)$, are tabulated for several values of n_2. The assumed values of n_1 and x are 1.342 and 50.0 π, respectively. The values of Q_a as obtained from these subroutines agree in four significant figures for n_2 up to 0.2. For $n_2 = 0.3$, a difference of one unit is noticeable in the fourth significant place. Thereafter, the difference increases very rapidly, and for $n_2 = 0.5$, the subroutine DAMIE returns negative values for Q_a.

A definite guideline for prohibiting the use of DAMIE subroutine cannot be easily established as the amount of absolute error propagated depends both upon $n_2 x$ and x. It depends upon x because the number of terms required for the convergence of the series depends upon x. In the above example, where $x = 157.08$, the results of DAMIE and DBMIE show significant differences for $n_2 x > 47.1$. On the other hand, for another case for which m was assumed to be $1.70 - 1.84\,i$, the results obtained with DAMIE and DBMIE were found to agree within six significant figures for x up to 40.0, i.e., $n_2 x = 73.6$.

The computations of the function $W_n(x)$ which always has a real argument, are straightforward. The problem of the propagation of error is not serious if all the basic arithmetic is done in double precision.

3.3 π_n and τ_n

The phase functions $\pi_n(\mu)$ and $\tau_n(\mu)$ given by eqs. (29) and (30) can be com

after making use of the following recurrence relationship based on the properties of the Legendre polynomials and their derivatives:

$$\pi_n(\mu) = \frac{2n-1}{n-1}\,\mu\,\pi_{n-1}(\mu) - \frac{n}{n-1}\,\pi_{n-2}(\mu) \quad , \tag{47}$$

and $$\tau_n(\mu) = n\,\mu\,\pi_n(\mu) - (n+1)\,\pi_{n-1}(\mu) \quad , \tag{48}$$

where

$$\left.\begin{aligned} \pi_0(\mu) &= 0 \quad , \\ \pi_1(\mu) &= 1 \quad , \\ \tau_0(\mu) &= 0 \quad , \\ \text{and} \quad \tau_1(\mu) &= \mu \quad . \end{aligned}\right\} \tag{49}$$

The straightforward upward recurrence poses no serious problem. The absolute error increases by a factor of about 10^6 as values $\pi_{3000}(\mu)$ and $\tau_{3000}(\mu)$ are obtained from those of $\pi_0(\mu)$, etc., given by eq. (49). Since this is a loss of six to seven significant figures, it is again necessary to carry out basic arithmetic in double precision.

The values of $\pi_n(1)$ and $\tau_n(1)$ obtained after making use of eqs. (47), (48), and (49), can be checked against those obtained from the following very simple expressions for the same (Van de Hulst, 1957):

$$\tau_n(1) = \tau_n(1) = \tfrac{1}{2}\,n\,(n+1) \tag{50}$$

For $\theta = 90^\circ$, i.e., $\mu = 0$, it can be shown that

$$\left.\begin{aligned} \pi_n(0) &= 0 \quad \text{(if } n \text{ is even)} \\ \text{and} \quad \pi_n(0) &= \frac{2(-1)^{(n-1)/2}\,\Gamma\left(\frac{n}{2}+1\right)}{\Gamma\left(\frac{n}{2}+\frac{1}{2}\right)\,\Gamma(1/2)} \quad \text{(if } n \text{ is odd)} \quad . \end{aligned}\right\} \tag{51}$$

The values of gamma functions for large values of n can be found in one of the tables in the series of Mathematical Tables published by the U. S. National Bureau of Standards (1951).

If the values of $\pi_n(\mu)$ and $\tau_n(\mu)$ for $\mu > 0$ have been already computed, it is not necessary to compute those of $\pi_n(-\mu)$ and $\tau_n(-\mu)$ as was done by Gumprecht and Sliepcevich (1951). Instead, one can make use of the following relationships:

$$\pi_n(-\mu) = (-1)^{n-1}\,\pi_n(\mu) \tag{52}$$

and

$$\tau_n(-\mu) = (-1)^n\,\tau_n(\mu) \quad . \tag{53}$$

These relationships can be derived easily by making use of the following well-known properties of the Legendre polynomials:

$$P_n(-\mu) = (-1)^n P_n(\mu) \quad , \quad (\mu \geq 0) \tag{54}$$

and

$$P'_n(-\mu) = (-1)^{n-1} P'_n(\mu) \quad , \quad (\mu \geq 0) \tag{55}$$

where P'_n represents the derivative of P_n with respect to μ.

However, it appears that the above mentioned properties have gone unnoticed in this particular field. Van de Hulst (1957) has not given them explicitly, even though he does make use of this relationship for $\mu = -1$ at several places. He has not commented on the tables of Gumprecht and Sliepcevich (1951) who have unnecessarily extended the length of the tables. The later researchers (e.g., Giese, 1961 ; Deirmendjian and Clasen, 1962 ; Giese, et al, 1962 ; Cunningham, et al, 1966 ; and Kattawar and Plass, 1967) do not refer to this relationship either. The advantages of making use of these relationships should be more evident after studying the published data of various authors, and after recognizing the ease of programming for a supplementary angle (appendices II and III).

IV. DETAILS ABOUT THE SUBROUTINES

4.1 General

Two subroutines, called DAMIE and DBMIE, are written with the purpose of helping the user to obtain parameters of the radiation scattered by a sphere of a given size. The actual FORTRAN statements (FORTRAN IV language) are listed in appendices II and III respectively. A survey of the literature showed that the user is likely to be interested in computing the radiation field for the value of the size parameter $x\ (= 2\pi r/\lambda) \sim 10^3$, and possibly for about the same number of directions of scattering. Such demands, along with the necessity of carrying out all computations in double precision, are bound to tax the storage capacities of any computer. In order to avoid any use of tapes or disks, it was decided to use the storage very sparingly. As mentioned in earlier sections, the main difference between the two subroutines is the manner in which one of the functions is evaluated. In DAMIE, the function $A_n(mx)$ is evaluated using an upward recurrence scheme, while in DBMIE a downward recurrence procedure has been used.

The CALL statement for either subroutine requires the following parameters in the order listed:

x: size parameter of the scattering sphere, $x > 0$.

n_1: real part of the refractive index of the material of the sphere; $n_1 > 1.0$ because the present subroutines are not tested for values of $n_1 \leq 1.0$;

TABLE 3

Some symbols as used in the text and their equivalence in DAMIE and DBMIE subroutines.

TEXT	SUBROUTINES
x	X
n_1	RFR
n_2	RFI
θ	THETD(J)
Q_e	QEXT
Q_s	QSCAT
$\overline{\cos\theta} \cdot Q_s$	CTBRQS
$M_2(\theta)$	ELTRMX (1,J,1)
$M_1(\theta)$	ELTRMX (2,J,1)
$S_{21}(\theta)$	ELTRMX (3,J,1)
$D_{21}(\theta)$	ELTRMX (4,J,1)
$M_2(180-\theta)$	ELTRMX (1,J,2)
$M_1(180-\theta)$	ELTRMX (2,J,2)
$S_{21}(180-\theta)$	ELTRMX (3,J,2)
$D_{21}(180-\theta)$	ELTRMX (4,J,2)
m	RF
W_{n-2} , W_{n-1} , W_n	WM1 , WFN(1) , WFN(2)
A_{n-1} , A_n	ACAP(1) , ACAP(2) ; ACAP(N) in DBMIE .
a_n , a_{n-1}	FNA, FNAP
b_n , b_{n-1}	FNB, FNBP
π_{n-2} , π_{n-1} , π_n	PI(1,J) , PI(2,J) , PI(3,J)
τ_{n-2} , τ_{n-1} , τ_n	TAU(1,J) , TAU(2,J) , TAU(3,J)

n_2: imaginary part of the refractive index of the material of the sphere, $n_2 \geq 0.0$;

θ: Angles between the directions of the scattered radiations and direction of the incident radiation for which computations are required; θ should be entered in degrees and its value should not exceed 90.0^0.

JX: Total number of θ's . Its value should not exceed 100 unless the dimensions in all related statements are appropriately changed . It must be greater than one.

It should be noted that the subroutines do not check for the conditions for x, n_1 and n_2. If the conditions are not satisfied, the subroutines are likely to return meaningless results.

The following quantities are returned by the subroutines: Q_e: eq. (31); Q_s: eq. (32); $\overline{\cos\theta} \cdot Q_s$: eq. (34); and the elements of the transformation matrix $\underline{F}$ (eq. 14) for all values of θ as well as $180 - \theta$ (Table 3).

For the proper functioning of the subroutines, it is necessary to declare the following as REAL * 8 or as DOUBLE PRECISION in the main program:

X, RFR, RFI, QEXT, QSCAT, CTBRQS, THETD(100), and ELTRMX (4,100,2).

After studying the subroutines in conjunction with Table 3, the following should be evident:

1. For a given n, the values of the functions W_n, π_n, a_n and b_n are stored only if they are required at a later stage.
2. The above statement also applies to the function A_n which is computed in DAMIE by upward recurrence.

3. The real and imaginary parts of a complex quantity are obtained by means of EQUIVALENCE.

4. Storage for the elements of the transformation matrix (which are the quantities computed at the very end) is first utilized to store the values of the complex amplitudes S_1 and S_2.

5. All the series are terminated when $|a_n|^2 + |b_n|^2 < 10^{-14}$.

4.2 Timing:

Average time t in seconds, which the subroutine DAMIE takes to return the values of the four elements of the transformation matrix $\underline{F}$ for 182 values of θ, as well as the values of Q_e, Q_s and $\overline{\cos\theta} \cdot Q_s$, is given in Table 4 for several values of x. The computing facility used was an IBM System/360 Model 50 with FORTRAN IV H compiler at OPT = 0.

From the data presented in Table 4, it can be seen that the subroutine becomes more efficient with the increase of x in the range 1.0 to 100.0, since a ten-fold increase in x does not result in a ten-fold increase in t. It may be noted that the most time is consumed in computations of a_n and b_n, and hence, the subroutine will become less and less efficient as the computations are called for fewer and fewer values of θ.

Because of the use of downward recurrence procedure, the average time t for the subroutine DBMIE depends upon the values of n_1, n_2, and x. However, in several cases for which the timing tests were run, DBMIE was found to take on the average of 10 to 20% more time than DAMIE.

TABLE 4

Average time (t in seconds) which the subroutine DAMIE takes to return values of four elements of the transformation matrix for 182 values of θ, as well as values of Q_e, Q_s and $\overline{\cos\theta} \cdot Q_s$ for a sphere having size parameter x. Computing facility used: IBM System/360 Model 50, FORTRAN IV H compiler OPT = 0.

x	t in seconds
1.0	2.0
10.0	6.5
100.0	39.5
1000.0	350.

The output given in the appendices IV and V was obtained using FORTRAN IV G compiler, OPT = 0. This compiler seems to be more efficient for these subroutines as the computations for $x = 1571$ take about 300 seconds only.

4.3 Storage Requirements:

DAMIE : 15,474 bytes

DBMIE : 127,340 bytes

4.4 Accuracy:

Because of the nature of the problem, it is not possible to make a positive statement about the reliability of a given value. An overall figure of reliability of the first six to seven significant figures seems to be quite reasonable.

It may be added that these subroutines were used to compute the characteristics of the radiation field for several cases for which the published as well as unpublished numerical values were available, (e.g., Gumprecht and Sliepcevich, 1953; Van de Hulst, 1957; Giese, 1961; Diermendjian and Clasen, 1962; Giese, et al, 1962; Irvine, 1963; Cunningham, et al, 1966; Plass, 1966; and Kattawar and Plass, 1967). As mentioned earlier, most of these results were obtained using single precision arithmetic and x up to 400 only. The results obtained using these subroutines and those obtained by the earlier authors were found to agree within the limits claimed by the various authors.

For a large sphere made of material with a refractive index of $1.342 - 0.0\,i$, Q_s, the efficiency factor for scattering, can be calculated from the following "best" formula suggested by Van de Hulst (1957, pp. 264, 265).

$$Q_s = 2.0 - 7.680 \sin(0.684\,x)\ \ 1.84\,x^{-(2/3)} + \text{"ripple"} \qquad (56)$$

For $x = 500.0\,\pi$ (a case for which the results are presented in Appendix IV) the second term is zero, and hence Q_s can be computed using the remaining three terms only. The value of Q_s so obtained is 2.0136 + "ripple", which compares very well with the value of 2.0129 given in the Appendix.

4.5 Sample Problem:

The FORTRAN statements for a sample problem with which one calls the DAMIE subroutine are listed in Appendix I. The same listing can be used for calling DBMIE after changing the CALL DAMIE ----- to CALL DBMIE ----- in the statement numbered DSMIE 036.

The sample problem program reads in the values of n_1, n_2, x, JX (number of x values for which computations are required), AJX (interval Δx) and MMM. All the values except that of integer JX should be punched on the card in double precision format. If the last quantity (MMM) is equal to zero, the value of x remains unchanged. If MMM $\neq$ 0, x is changed to $2\pi x/(\text{MMM} \times 10^{-3})$. The TIME subroutine called at stages DSMIE 035 and DSMIE 039 helps to read the clock in milliseconds. If this subroutine is not available, these two statements, as well as the statements numbered DSMIE 019, DSMIE 040, and DSMIE 065 can be omitted.

Besides other related information and the quantities Q_e, Q_s, Q_a and $\overline{\cos\theta}$, the sample program also prints out the values of all the four elements of the matrix $\underline{F}$ as well as the intensity and the degree of polarization of the radiation scattered by a sphere illuminated with natural light.

V. CONCLUSION

The problem of numerical evaluation of the characteristics of the electromagnetic radiation scattered by a sphere (Mie scattering) has been analyzed in great detail in previous sections. It has been shown that the question of numerical instability (Kattawar and Plass, 1967) when computing one of the several functions by upward recurrence is an important one indeed if the sphere is partly-absorbing, and is of moderate size (size parameter $x \sim 100$ or more). Following this, two subroutines are presented.

The subroutine DBMIE is expected to give reliable results under all foreseeable conditions. On the other hand, the subroutine DAMIE is expected to give reliable results only when the moderate and large spheres are made of non-absorbing material. For a partly-absorbing sphere, the results obtained using DAMIE can be expected to be reliable only if the absorption and the size parameters are of moderate magnitude ($n_2 x \sim 80$, $x \sim 100$). The present series of tests are insufficient to lay down more specific criterion for limiting the use of DAMIE. Apart from providing a concise subroutine which can be used in several cases with great ease, the other reason for providing DAMIE is to permit the user to check the reliability of some of the vast amount of published and unpublished numerical data on this subject.

These subroutines are fairly efficient from the point of view of the storage requirements and timing considerations. However, since they are basically general purpose subroutines, more efficient versions can be obtained after appropriate changes. One may be able to relax some of the stringent criteria in some limited applications, and after checks in extreme cases, one may be able to carry out all the basic arithmetic in single precision.

In the end, it should be added that though these subroutines are checked in several alternate ways, they should be still be treated as experimental ones. Due precaution must be exercised before accepting some unusual results in new cases.

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FOOTNOTE

Very recently, Fahlen and Bryant (1968) have reported some results for the radiation back-scattered by a large sphere (x = 3000,0).

Fahlen, T.S. and H. C. Bryant, Journ. Opt. Soc. Amer., Vol. 58, 1968, pp. 304-310.

APPENDIX I

Listing of Sample Problem

```
// FULL FORTHCLG,PARM.FORT='MAP'                                        DSMIE002
//FORT.SYSIN DD *                                                      DSMIE003
C     CONTAINS MIE SUBROUTINES..                                       DSMIE004
   10 FORMAT(3D15.5,3I5,D15.5,I10)                                     DSMIE005
   12 FORMAT(10E5.5)                                                   DSMIE006
   20 FORMAT(1H1)                                                      DSMIE007
   25 FORMAT(///10X,'ELEMENTS OF THE TRANSFORMATION MATRIX FOR A SPHERE DSMIE008
     1 WITH SIZE PARAMETER = ',F15.5)                                  DSMIE009
   30 FORMAT(//10X,'REFRACTIVE INDEX, REAL = ',D15.5,T60,'IMAGINARY',  DSMIE010
     1D15.5/)                                                          DSMIE011
   35 FORMAT(1X,'ANGLE',T17,'M SUB 2 ',T31,' M SUB 1',T46,' S SUB 21', DSMIE012
     1T61,'S SUB 22',T76,'INTENSITY',T91,'POLARIZATION'//)             DSMIE013
   40 FORMAT(F10.4,5E15.5,E15.4)                                       DSMIE014
   45 FORMAT(//10X,' EFFICIENCY FACTOR FOR EXTINCTION',E15.6)          DSMIE015
   50 FORMAT(//10X,' EFFICIENCY FACTOR FOR SCATTERING',E15.6)          DSMIE016
   55 FORMAT(//10X,' EFFICIENCY FACTOR FOR ABSORPTION',E15.6)          DSMIE017
   60 FORMAT(//10X,' ASYMMETRY FACTOR',E15.6///)                       DSMIE018
   70 FORMAT(//10X,' TOTAL TIME FOR THIS CASE IN SECONDS = ',F15.3///) DSMIE019
      DIMENSION XEN(401),X(401),QEXT(401),QSCAT(401),QABS(401),ELEMX(4,DSMIE020
      DIMENSION ALAM(100),ELEMS(4,4,401)                               DSMIE021
      DIMENSION AINT(3,2),POLRE(3,2)                                   DSMIE022
      PI = 3.14159265358979323D0                                       DSMIE023
   80 READ(5,10) XEN,XST,X,ALAM,MMM                                    DSMIE024
      IF (XST .LT. 0.0D0) GO TO 1000                                   DSMIE025
      READ(5,12) THETA                                                 DSMIE026
      IF (MMM .GT. 0) GO TO 90                                         DSMIE027
C     MMM = 0,THE VALUE OF X IS TO BE CHANGED,OTHERWISE, X IS          DSMIE028
C     UNCHANGED FOR EACH VALUE OF WAVELENGTH..                         DSMIE029
      ALAM = MMM * 1.0D-4                                              DSMIE030
      X = 2.0D0 * PI * X / ALAM                                        DSMIE031
   90 DO 100 J = 2,3                                                   DSMIE032
      THETA(J) = (J-1) * STX + THETA(1)                                DSMIE033
  100 CONTINUE                                                         DSMIE034
      THETA = THETA(1)                                                 DSMIE035
      CALL MIE( X,XEN,XST,THETA,2,ELEM,QSCAT,QTOAS,ELEMX)              DSMIE036
      QABS = QEXT - QSCAT                                              DSMIE037
      ALBEDO = QSCAT/QEXT                                              DSMIE038
      FACT = PI*X*X                                                    DSMIE039
      EFFE = QEXT * FACT * 1.0E-4                                      DSMIE040
      DO 130 K = 1,2                                                   DSMIE041
      DO 130 J = 1,2                                                   DSMIE042
      AINT(J,K) = ELEMX(1,J,K) + ELEMX(2,J,K)                          DSMIE043
      POLRE(J,K) = ELEMX(2,J,K) - ELEMX(3,J,K)/AINT(J,K)               DSMIE044
      AINT(J,K) = 0.5 * AINT(J,K)                                      DSMIE045
  130 CONTINUE                                                         DSMIE046
      WRITE(6,20)                                                      DSMIE047
      WRITE(6,25) X                                                    DSMIE048
      WRITE (6,40) XEN,XST                                             DSMIE049
      WRITE (6,35)                                                     DSMIE050
      WRITE(6,40)(THETA(J),(ELEMX(I,J,1),I=1,4),AINT(J,1),POLRE(J,1),  DSMIE051
     1J = 1,2)                                                         DSMIE052
      DO 150 J = 1,2                                                   DSMIE053
      THETA(J) = 180.0D0 - THETA(J)                                    DSMIE054
  150 CONTINUE                                                         DSMIE055
      JMM = 2K - 1                                                     DSMIE056
      DO 210 J = 1,2MM                                                 DSMIE057
      JJ = 2K - J                                                      DSMIE058
      WRITE(6,40)THETA(JJ),(ELEMX(I,JJ,2),I=1,4),AINT(JJ,2),POLRE(JJ,2)DSMIE059
  210 CONTINUE                                                         DSMIE060
      WRITE(6,45) QEXT                                                 DSMIE061
```



```
      WRITE(6,50) QSCAT                                                DSMIE062
      WRITE(6,55) QABS                                                 DSMIE063
      WRITE(6,60) ALBEDO                                               DSMIE064
      WRITE(6,70) ETIME                                                DSMIE065
      WRITE(6,20)                                                      DSMIE066
      GO TO 80                                                         DSMIE067
 1000 RETURN                                                           DSMIE068
      END                                                              DSMIE069
```

APPENDIX II

Listing of DAMIE Subroutine

```
0001          SUBROUTINE DAMIE (X,NFK,RFI,THETD,JX,QEXT,QSCAT,CTORQS,ELTMX1   DAMIE001
C             SUBROUTINE FOR COMPUTING THE PARAMETERS OF THE ELECTROMAGNETIC  DAMIE002
C             RADIATION SCATTERED BY A SPHERE. THIS SUBROUTINE CARRIES OUT ALL DAMIE003
C             COMPUTATIONS IN DOUBLE PRECISION ARITHMETIC.                    DAMIE004
C             X : SIZE PARAMETER OF THE SPHERE,I.E. 2 * PI * RADIUS OF THE SPHERE/WAVDAMIE005
C             WAVELENGTH OF THE INCIDENT RADIATION).                          DAMIE006
C             RFI: REFRACTIVE INDEX OF THE MATERIAL OF THE SPHERE.COMPLEX     DAMIE007
C             QUANTITY..FORM: ( REAL  -  I * IMAG )                           DAMIE008
C             THIS SUBROUTINE WILL NOT GIVE RELIABLE RESULTS IF RFI * X IS    DAMIE009
C             GREATER THAN 80.0.IF SO, A MESSAGE IS PRINTED AND THE           DAMIE010
C             COMPUTATIONS ARE CONTINUED.  SUGGEST USE DAMIE..                DAMIE011
C             THETD(I): ANGLE IN DEGREES BETWEEN THE DIRECTIONS OF THE INCIDENT DAMIE012
C             AND THE SCATTERED RADIATION. THETD(I) IS < OR = 90.0            DAMIE013
C             IF THETD(I) SHOULD HAPPEN TO BE GREATER THAN 90.0, ENTER WITH   DAMIE014
C             SUPPLEMENTARY VALUE; SEE COMMENTS BELOW ON ELTMX..              DAMIE015
C             JX: TOTAL NUMBER OF THETD FOR WHICH THE COMPUTATIONS ARE        DAMIE016
C             REQUIRED. JX SHOULD NOT EXCEED 100 UNLESS THE DIMENSIONS        DAMIE017
C             STATEMENTS ARE APPROPRIATELY MODIFIED.                          DAMIE018
C             MAIN PROGRAM SHOULD ALSO HAVE REAL*8 THETD(100),ELTMX1(4,100,2). DAMIE019
C             THE DEFINITIONS FOR THE FOLLOWING SYMBOLS CAN BE FOUND IN ' LIGHT DAMIE020
C             SCATTERING BY SMALL PARTICLES,H.C. VAN DE HULST, JOHN WILEY & SONSDAMIE021
C             , INC., NEW YORK, 1957 ' .                                      DAMIE022
C             QEXT: EFFICIENCY FACTOR FOR EXTINCTION,VAN DE HULST,P.14 & 127. DAMIE023
C             QSCAT: EFFICIENCY FACTOR FOR SCATTERING,VAN DE HULST,P.14 & 127. DAMIE024
C             GTORQS: AVERAGE(COSINE THETA) * QSCAT,VAN DE HULST,P.128.       DAMIE025
C             ELTMX1(I,J,K): ELEMENTS OF THE TRANSFORMATION MATRIX F,VAN DE HULDAMIE026
C             ST,P,3RD ED. P 125. I = 1: ELEMENT M SUB 2..I = 2: ELEMENT M SUB 1..DAMIE027
C             I = 3: ELEMENT S SUB 21.. I = 4: ELEMENT D SUB 21..             DAMIE028
C             ELTMX1(J,1) REPRESENTS THE ITH ELEMENT OF THE MATRIX FOR        DAMIE029
C             THE ANGLE THETD(J).. ELTMX1(J,2) REPRESENTS THE ITH ELEMENT     DAMIE030
C             OF THE MATRIX FOR THE ANGLE 180.0 - THETD(J) ..                 DAMIE031
C                                                                             DAMIE032
      1 FORMAT(1H0,' THE VALUE OF THE SCATTERING ANGLE IS GREATER THAN        DAMIE033
     1 90.0 DEGREES.. IT IS ',G15.4)                                          DAMIE034
0203  2 FORMAT(1H0,' PLEASE READ COMMENTS.')                                  DAMIE035
0204  3 FORMAT(1H0,'THE VALUE OF THE ARGUMENT JX IS GREATER THAN 100')        DAMIE036
0205  4 FORMAT(1H0,' THE VALUE OF RFI * X IS GREATER THAN 80.0.IT IS',       DAMIE037
     1 G15.4)                                                                 DAMIE038
0046          REAL*8 X,WX,WFK,WFI,QEXT,QSCAT,TES1,TAI1,TWI1,TCL1              DAMIE039
0047          REAL*8 THI1,EL1,GLTORQS                                         DAMIE040
0048          REAL*8 ELTMX1(4,100,2),PII,DJX,TAI3,TWI3,CSTHI1(100),SI2THI1(100DAMIE041
     1,THETD(100)                                                             DAMIE042
0049          COMPLEX*16 WF,WWF,WWF2,WM1,FNA,FNB,TC1,TC2,WFNI1,ALAPI1         DAMIE043
0050          COMPLEX*16 FNAP,FNBP                                            DAMIE044
C             TAI1: REAL PART OF WFNI1.. TAI2: IMAGINARY PART OF WFNI1..      DAMIE045
C             TAI3: REAL PART OF WFNI2..TAI4: IMAGINARY PART OF WFNI2.        DAMIE046
C             TAI1: REAL PART OF FNA.. TBI2: IMAGINARY PART OF FNA..          DAMIE047
C             TCI1: REAL PART OF FNB.. TCI2: IMAGINARY PART OF FNB..          DAMIE048
C             TWI1: REAL PART OF FNAP..TWI2: IMAGINARY PART OF FNAP.          DAMIE049
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C     TE121: REAL PART OF FNNP., TE122: IMAGINARY PART OF FNNP.
C     FNAP & FNNP ARE THE PRECEDING VALUES OF FNA & FNN RESPECTIVELY.
      EQUIVALENCE (WFNE11,TA111),(FNA,TE111),(FNN,TE112)
      EQUIVALENCE (FNAP,TE121),(FNNP,TE122)
      IF ( Z .EQ. 1.0D 0 ) GO TO 20
      WRITE(6,7)
      WRITE(6,8)
      CALL EXIT
   20 RF = DLMPLX(RFR,-RFI)
      RR1 = 1.0D0/RF
      RX = 1.3D0/8
      RRRX = RR1 * RX
      DO 30 J = 1,28
      IF ( THETU(J) .LE. 0.000 ) THETU(J) = DABS(THETU(J))
      IF ( THETU(J) .GT. 0.000 ) GO TO 23
      CSTHE(J) = 1.000
      SIZTHE(J) = 0.000
      GO TO 30
   23 IF ( THETU(J) .GE. 90.000 ) GO TO 25
      TE11 = 3.1415926535897932 * THETU(J)/180.000
      CSTHE(J) = DCOS(TE11)
      SIZTHE(J) = 1.000 - CSTHE(J)**2
      GO TO 30
   25 IF ( THETU(J) .GT. 90.000 ) GO TO 28
      CSTHE(J) = 0.000
      SIZTHE(J) = 1.000
      GO TO 30
   28 WRITE(6,9) THETU(J)
      WRITE(6,8)
      CALL EXIT
   30 CONTINUE
      DO 35 J = 1,28
      P111,J) = 0.000
      P112,J) = 1.000
      TAU1(J) = 0.000
      TAU2(J) = CSTHE(J)
   35 CONTINUE
      TE13 = DCOS(E3)
      TE21 = DSIN(E3)
      WN1 = DLMPLX( TE13,-TE23)
      WFN11 = DLMPLX(TE13,TE23)
      WFN12 = RX * WFN11 - WN1
      TE11 = RR1 * X
      IF ( TE11 .GT. 80.000 ) GO TO 40
      TE11 = 3.500 * DEXP(TE11)
      TE12 = 0.2500/TE11
      TE13 = TE11 + TE12
      TE14 = TE11 - TE12
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      TE31 = TE28**2
      TE21 = TE11 * TE23
      TE11 = TE31
      TE31 = RFR * R
      TE41 = DSIN(TE31)
      TE32 = DCOS(TE31)
      TE33 = TE13 + TE41**2
      TE34 = TE21 * TE41
      AC4P(I) = DLMPLX(TE32,TE21/TE11)
      GO TO 53
   40 AC4P(I) = DLMPLX( 0.000,1.000 )
      WRITE(6,43) TE11
      WRITE(6,8)
   50 AC4P(J) = - RRFR + (1.000/RNFR - AC4P(1))
      TL1 = AC4P(J) * RR * RX
      TL2 = AC4P(J) * RR * RX
      FNA = (TL1*TA111 - TA112)/(TL1*WFN12 - WFN11)
      FNN = ( TL2*TA111 - TA112)/(TL2 * WFN12 - WFN11)
      FNAP = FNA
      FNNP = FNN
      TE11 = 1.0D0
      ENTR (REAL TO THE STATEMENT NUMBER 90. ELTRM(I,J,K) HAS THE
C     SUBSCRIPTED MEANING:
C     ELTRM(I,J,K1) REAL PART OF THE FIRST COMPLEX AMPLITUDE.
C     ELTRM(I,J,K2) IMAGINARY PART OF THE FIRST COMPLEX AMPLITUDE.
C     ELTRM(I,J,K1) REAL PART OF THE SECOND COMPLEX AMPLITUDE.
C     ELTRM(I,J,K2) IMAGINARY PART OF THE SECOND COMPLEX AMPLITUDE.
C     K = 1 FOR THETU(J) AND K = 2 FOR 180.0 - THETU(J)
C     DEFINITION OF THE COMPLEX AMPLITUDE: VAN DE HULST,P.125.
      TU11 = TE11 * TU11
      TU21 = TL11 * TU21
      TL11 = TE11 * TL11
      TL21 = TE11 * TL21
      DO 60 J = 1,28
      ELTRM(I,J,1) = TU11 * PI12,J) + TL11 * TAU2,J)
      ELTRM(I,J,2) = TU21 * PI12,J) + TL21 * TAU2,J)
      ELTRM(I,J,3) = TL11 * PI12,J) + TU11 * TAU2,J)
      ELTRM(I,J,4) = TL21 * PI12,J) + TU21 * TAU2,J)
      ELTRM(I,J,1) = TU11 * PI12,J) - TL11 * TAU2,J)
      ELTRM(I,J,2) = TU21 * PI12,J) - TL21 * TAU2,J)
      ELTRM(I,J,3) = TL11 * PI12,J) - TU11 * TAU2,J)
      ELTRM(I,J,4) = TL21 * PI12,J) - TU21 * TAU2,J)
   60 CONTINUE
      QEXT = 2.000 * ( TE11 + TL11 )
      QSCAT =(TU11**2 + TU21**2 + TL11**2 + TL21**2)/0.7500
      CTBAR = 0.000
      N = 2
   65 TE11 = 2** - 1
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      TEZI = N - 1                                                      DAMIE149
      TEII = Z * N + 1                                                  DAMIE146
      DO 70 J = 1,2N                                                    DAMIE147
      PIE(N,JI = EZEII*PIEZ,JI+ELSTHETZI-N*PIII,JI)/TEZI                DAMIE148
      TAUE(N,JI = CSTHETZI*EPIE(N,JI-PIEI,JII-TEII*SITHETZI*PIEZ,JI+    DAMIE149
     1 TAUE(I,JI                                                        DAMIE150
70 CONTINUE                                                             DAMIE151
      WMI = WFNII                                                       DAMIE152
      WFNII = WFNEZI                                                    DAMIE153
      WFNEZI = TEII*RA*WFNII - WMI                                      DAMIE154
      ALAPII = ALAPEZI                                                  DAMIE155
      ALAPEZI = -U + RNIR + EI.UUU/IN*RHFA-ALAPIIII                     DAMIE156
      FCI = ALAPIZI*RMI + R*RX                                          DAMIE157
      FCZ = ALAPEZI*RI  + R*RX                                          DAMIE158
      FNA = EELI*TAEII-TAEIII/EICI*WFNEZI - WFNEII                      DAMIE159
      FNB = EELZ*TAEII-TAEIII/EICZ*WFNEZI - WFNEII                      DAMIE160
      IEEU = N                                                          DAMIE161
      EIAI = EEEZIIIII*TIZII                                            DAMIE162
      EIZI = EEEZI*IIIAI + I.UUUIIZIIAI                                 DAMIE163
      CIAIZI = CEIMAI + EIZI*ETUIII*TUIII + TDIZI*TUEZI + TECIII*TLCII + DAMIE164
     1 TEEZI*TLCZII + EIAI*ETUIII*TECII + TDIZI*TECZII                  DAMIE165
      UIAI = UEAI + EEII*ETBIII*TCIIII                                  DAMIE166
      EIAI = TDEII**Z + TECZI**Z + TLCIII**Z + TLCZI**2                 DAMIE167
      UVCAI = UUCAI + IEII *EIAI                                        DAMIE168
      TIZI = 4*EIAI*EI                                                  DAMIE169
      TIII = TEIII/TIZI                                                 DAMIE170
      R = IAEZI*Z                                                       DAMIE171
      DO 80 J = 1,2N                                                    DAMIE172
      CLIAMAII,J,II=CLIAMAII,J,II+TEII*(TUEII*PIE3,JI+TCEII*TAUE3,JII   DAMIE173
      CLIAMAIZ,J,II=CLIAMAIZ,J,II+TEII*(TDEZI*PIE3,JI+TCIZI*TAUE3,JII   DAMIE174
      CLIAMAE3,J,II=CLIAMAI3,J,II+TEII*(T_CII3*PIE3,JI+TDCIII*TAUE3,JII DAMIE175
      CLIAMAI4,J,II=CLIAMAI4,J,II+TEII*(T_CI3*PIE3,JI+TDCIZI*TAUE3,JII  DAMIE176
      IF ( R .GE. 4 ) GO TO 75                                          DAMIE177
      CLIAMAII,J,ZI=CLIAMAII,J,ZI+TEII*(TDEII*PIE3,JI-TCIII*TAUE3,JII   DAMIE178
      CLIAMAIZ,J,ZI=CLIAMAIZ,J,ZI+TEII*(TDCZI*PIE3,JI-TCCZI*TAUE3,JII   DAMIE179
      CLIAMAI3,J,ZI=CLIAMAI3,J,ZI+TEII*(TCCIII*PIE3,JI-TDEII*TAUE3,JII  DAMIE180
      CLIAMAI4,J,ZI=CLIAMAI4,J,ZI+TEII*(TCCZI*PIE3,JI-TDEZI*TAUE3,JII   DAMIE181
      GO TO 80                                                          DAMIE182
75 CLIAMAII,J,ZI=CLIAMAII,J,ZI+TEII*(1-TDEII*PIE3,JI+TCEIII*TAUE3,JII  DAMIE183
      CLIAMAIZ,J,ZI=CLIAMAIZ,J,ZI+TEII*(1-TDCZI*PIE3,JI+TCCZI*TAUE3,JII DAMIE184
      CLIAMAI3,J,ZI=CLIAMAI3,J,ZI+TEII*(1-TCCIII*PIE3,JI+TDEII*TAUE3,JII DAMIE185
      CLIAMAI4,J,ZI=CLIAMAI4,J,ZI+TEII*(1-TCCZI*PIE3,JI+TDEZI*TAUE3,JII DAMIE186
80 CONTINUE                                                             DAMIE187
      IF( EIAI .LT. 1.UU-14 ) GO TO 100                                 DAMIE188
      N = N + 1                                                         DAMIE189
      DO 90 J = 1,2N                                                    DAMIE190
      PIEI,JI = PIEZ,JI                                                 DAMIE191
      PIEZ,JI = PIE3,JI                                                 DAMIE192
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      TAU11,J1 = TAU12,J1                                               DAMIE193
      TAU12,J1 = TAU13,J1                                               DAMIE194
   40 CONTINUE                                                          DAMIE195
      FNAP = FNA                                                        DAMIE196
      FNBP = FNB                                                        DAMIE197
      GO TO 50                                                          DAMIE198
  100 DO 120 J = 1, 20                                                  DAMIE199
      DO 120 K = 1,2                                                    DAMIE200
      DO 120 I = 1,4                                                    DAMIE201
      TE33 = ELTHMM11,J,K1                                              DAMIE202
  120 CONTINUE                                                          DAMIE203
      ELTHMM12,J,K1 = TE33**2 + TE23**2                                 DAMIE204
      ELTHMM13,J,K1 = TE33**2 + TE43**2                                 DAMIE205
      ELTHMM21,J,K1 = TE33*TE23 + TE23*TE43                             DAMIE206
      ELTHMM14,J,K1 = TE23*TE33 - TE43*TE13                             DAMIE207
  130 CONTINUE                                                          DAMIE208
      TE13 = ZCOMPLX * CONJG*                                           DAMIE209
      GEAT = GEAT + TE13                                                DAMIE210
      GSCAT = GSCAT + TE13                                              DAMIE211
      CTHMAS = CTHMAS + TE13                                            DAMIE212
C     THE DETAILED WORK THIS SUBROUTINE CAN BE FOUND IN THE FOLLOWING   DAMIE213
C     REPORT: " SUBROUTINES FOR COMPUTING THE PARAMETERS OF THE         DAMIE214
C     ELECTROMAGNETIC RADIATION SCATTERED BY A SPHERE " J.V. DAVE,      DAMIE215
C     I B M SCIENTIFIC CENTER, PALO ALTO , CALIFORNIA,                  DAMIE216
C     REPORT NO. 320-3237 , MAY 1968.                                   DAMIE217
      RETURN                                                            DAMIE218
      END                                                               DAMIE219
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 2004 SUBROUTINE DAMIE(X,REFR,REFI,THETD,JX,QEXT,QSCAT,CTHMAS,ELTHMAT   DAMI1
C     SUBROUTINE FOR COMPUTING THE PARAMETERS OF THE ELECTROMAGNETIC    DAMI1
C     RADIATION SCATTERED BY A SPHERE. THIS SUBROUTINE CARRIES OUT ALL  DAMI1
C     COMPUTATIONS IN DOUBLE PRECISION ARITHMETIC.                      DAMI1
C     THIS SUBROUTINE COMPUTES THE CAPITAL A FUNCTION BY MAKING USE OF  DAMI1
C     DOWNWARD RECURRENCE RELATIONSHIP.                                 DAMI1
C     X : SIZE PARAMETER OF THE SPHERE,( Z = PI * RADIUS OF THE SPHERE)/DAMI1
C     WAVELENGTH OF THE INCIDENT RADIATION).                            DAMI1
C     REFI: REFRACTIVE INDEX OF THE MATERIAL OF THE SPHERE.COMPLEX      DAMI1
C     QUANTITY.-FORM: ( REFR  -  I * REFI )                             DAMI1
C     THETD(J): ANGLE IN DEGREES BETWEEN THE DIRECTIONS OF THE INCIDENT DAMI1
C     AND THE SCATTERED RADIATION. THETD(J) IS < OR = 90.0              DAMI1
C     IF THETD(J) SHOULD HAPPEN TO BE GREATER THAN 90.0, ENTER WITH     DAMI1
C     SUPPLEMENTARY VALUE: SEE COMMENTS BELOW ON ELTHMX..               DAMI1
C     JX: TOTAL NUMBER OF THETD FOR WHICH THE COMPUTATIONS ARE          DAMI1
C     REQUIRED. JX SHOULD NOT EXCEED 100 UNLESS THE DIMENSION           DAMI1
C     STATEMENTS ARE APPROPRIATELY MODIFIED.                            DAMI1
C     MAIN PROGRAM SHOULD ALSO HAVE REAL*8 THETD(100),ELTHMX(4,100,2).  DAMI1
C     THE DETAILED FORM FOR THE FOLLOWING SYMBOLS CAN BE FOUND IN " LIGHT DAMI1
C     SCATTERING BY SMALL PARTICLES,H.C. VAN DE HULST, JOHN WILEY & SONS,DAMI1
C     , INC., NEW YORK, 1957 " .                                        DAMI1
C     QEXT: EFFICIENCY FACTOR FOR EXTINCTION,VAN DE HULST,P.14 & 127.   DAMI1
C     QSCAT: EFFICIENCY FACTOR FOR SCATTERING,VAN DE HULST,P.14 & 127.  DAMI1
C     GSCAT: ASYMMETRY FACTOR FOR SCATTERING,VAN DE HULST,P.128.        DAMI1
C     CTHMAS: AVERAGE(COSINE THETA) * QSCAT,VAN DE HULST,P.128.         DAMI1
C     ELTHMX(I,J,K): ELEMENTS OF THE TRANSFORMATION MATRIX F,VAN DE HUL DAMI1
C     STOKES PARAM. = 1/2. I = 1: ELEMENT M SUB 2..1 * 2: ELEMENT M SUB 1..DAMI1
C     I = 3: ELEMENT S SUB 21.. I = 4: ELEMENT D SUB 21..               DAMI1
C     ELTHMX(I,J,1) REPRESENTS THE ITH ELEMENT OF THE MATRIX FOR        DAMI1
C     THE ANGLE THETD(J).. ELTHMX(I,J,2) REPRESENTS THE ITH ELEMENT     DAMI1
C     OF THE MATRIX FOR THE ANGLE 180.0 - THETD(J) ..                   DAMI1
      IMPLICIT REAL*8 (A-H,O-Z)                                         DAMI1
 2214 FORMAT(1H ,' THE VALUE OF THE SCATTERING ANGLE IS GREATER THAN    DAMI1
     1 90.0 DEGREE.. IT IS ',D15.7)                                     DAMI1
 0003 FORMAT (1H1,' PLEASE READ COMMENTS.')                             DAMI1
 0054 FORMAT(1H1,'THE VALUE OF THE ARGUMENT IX IS GREATER THAN 100')    DAMI1
 0075 FORMAT(1H1,'THE UPPER LIMIT FOR NSAP IS NOT ENOUGH. SUGGEST SET   DAMI1
     1UP DETAILED OUTPUT AND MODIFY SUBROUTINE')                        DAMI1
 0016      REAL*8 X,XX,REFR,REFI,WRE1,WSCAT,TF1(3),TF2(3),TD(2),TC(2)   DAMI1
 0037      REAL*8 TU(2),TL(2),CTHMAS                                     DAMI1
 0038      REAL*8 ELTHMX(4,100,2),PI2,PI,3.1D0,TAU(3,100),CSTHT(100),SIZ(DAMI1
     1,THETD(100)                                                       DAMI1
 0079      COMPLEX*16 WF,WRE,XXFR,WM1,FNA,FNB,TCL,TL2,WFN(2),AQAPE(2000) DAMI1
 0010      COMPLEX*16 FNAP,FNBP                                          DAMI1
C          TF1(1): REAL PART OF WFN(1).. TF1(2): IMAGINARY PART OF WFN(1)DAMI1
C          TF2(1): REAL PART OF WFN(2)..TF2(2): IMAGINARY PART OF WFN(2).DAMI1
C          TD(1): REAL PART OF FNA.. TD(2): IMAGINARY PART OF FNA..       DAMI1
C          TC(1): REAL PART OF FNB.. TC(2): IMAGINARY PART OF FNB..       DAMI1
C          TU(1): REAL PART OF FNAP..TU(2) IMAGINARY PART OF FNAP.        DAMI1
C          TL(1): REAL PART OF FNBP.. TC(2): IMAGINARY PART OF FNBP.      DAMI1
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C     FNAP & FNAP ARE THE PRECEDING VALUES OF FNA & FNB RESPECTIVELY.
      SUBROUTINE CMPNII(WPNII,TAII,IFNA,TBII),(FNB,TCII))
      SUBROUTINE CMPNII(WAP,TWII),(FNBP,TEII))
      IF ( Z .EQ. 100 ) GO TO 20
      WRITE(6,7)
      WRITE(6,8)
      CALL EXIT
   20 WP = (0.,-1.)
      WNF = 1.000/XP
      RA = 1.000/R
      RNFR = WNF * RR
      TIII = (XP**2)*(RR**2 + WP**2)
      TIII = SQRT(TIII)
      WMI = 1.1000 * TIII
      IF ( WMI .LE. 0.999 ) GO TO 21
      WRITE(6,9)
      CALL EXIT
   21 WMI = TIII
      IF ( WMI .GT. 150 ) GO TO 22
      NMI = 150
      NMI2 = 135
   22 ACAPI(WMI + 1 ) = ( 0.000, 0.000 )
      DO 23 N = 1,NMI
      NN = NMI - N + 1
      ACAPI(NN) = (N+1) * RRFX - 1.000/((NN+1)*RNFX + ACAPI(NN+1))
   23 CONTINUE
      DO 50 J = 1,JX
      IF ( THETU(J) .LT. 0.000 ) THETD(J) = ABS(THETU(J))
      IF ( THETU(J) .GT. 0.000 ) GO TO 24
      CSTHT(J) = 1.000
      SIZTHT(J) = 0.000
      GO TO 50
   24 IF ( THETU(J) .GE. 90.000 ) GO TO 25
      TEJJ = ( 3.1415926535897932 * THETU(J))/180.000
      CSTHT(J) = DCOS(TEJJ)
      SIZTHT(J) = 1.000 - CSTHT(J)**2
      GO TO 50
   25 IF ( THETU(J) .GT. 90.000 ) GO TO 26
      CSTHT(J) = 0.000
      SIZTHT(J) = 1.000
      GO TO 50
   26 WRITE(6,27) THETU(J)
      WRITE(6,8)
      CALL EXIT
   50 CONTINUE
      DO 55 J = 1,JX
      PII(1,J) = 0.000
      PII(2,J) = 1.000
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      TAUI(1,J) = 0.000
      TAUI(2,J) = CSTHT(J)
   55 CONTINUE
      TII = DCOS(R)
      TI2 = DSIN(R)
      WMI = DCMPLX( TII,-TI2)
      WPNII = DCMPLX(TI2,TII)
      WPNI(2) = RR * WPNII - WMI
      TC1 = ACAPI(1) * RRP + RR
      TC2 = ACAPI(1) * RP + RR
      FNA = (TC1*TAII - TAIII/(TC1*WPNI(2) - WPNII)
      FNB = ( TC2*TAI(2) - TAI(1)/(TC2 * WPNI(2) - WPNII)
      FNAP = FNA
      FNBP = FNB
      TEII = 1.0000
C     FROM HERE TO THE STATEMENT NUMBER 90, ELTMMX(I,J,K) HAS THE
C     FOLLOWING MEANING:
C     ELTMMX(I,J,K): REAL PART OF THE FIRST COMPLEX AMPLITUDE.
C     ELTMMX(2,J,K): IMAGINARY PART  OF THE FIRST COMPLEX AMPLITUDE.
C     ELTMMX(3,J,K): REAL PART OF THE SECOND COMPLEX AMPLITUDE.
C     ELTMMX(4,J,K): IMAGINARY PART OF THE SECOND COMPLEX AMPLITUDE.
C     K = 1 : FOR THETU(J) AND K = 2 : FOR 180.0 - THETU(J)
C     DEFINITION OF THE COMPLEX AMPLITUDE: VAN DE HULST,P.125.
      TB(1) = TII * TWII
      TB(2) = TII * TWI2
      TC(1) = TII * TCII
      TC(2) = TII * TCI2
      DO 60 J = 1,JX
      ELTMM(1,J,1) = TB(1) * PII(2,J) + TC(2) * TAU(2,J)
      ELTMM(2,J,1) = TB(2) * PII(2,J) + TC(2) * TAU(2,J)
      ELTMM(3,J,1) = TC(1) * PII(2,J) + TB(1) * TAU(2,J)
      ELTMM(4,J,1) = TC(2) * PII(2,J) + TB(2) * TAU(2,J)
      ELTMM(1,J,2) = TB(1) * PII(2,J) - TC(1) * TAU(2,J)
      ELTMM(2,J,2) = TB(2) * PII(2,J) - TC(2) * TAU(2,J)
      ELTMM(3,J,2) = TC(1) * PII(2,J) - TB(1) * TAU(2,J)
      ELTMM(4,J,2) = TC(2) * PII(2,J) - TB(2) * TAU(2,J)
   60 CONTINUE
      QEXT = 2.000 * ( TB(1) + TC(1))
      QSCAT =(TB(1)**2 + TB(2)**2 + TC(1)**2 + TC(2)**2)/0.7500
      CFBAR = 0.000
      N = 2
   65 TEJJ = 2*N - 1
      TE2) = N - 1
      TE3) = 2 * N + 1
      DO 70 J = 1,JX
      PII3,J) = (TEJJ*PII2,J)*CSTHT(J)-N*PII1,J))/TE2)
      TAU(3,J) = CSTHT(J)*(PII3,J)-PII1,J))-TE3)*SIZTHT(J)**PII2,J)+
     1 TAU(1,J)
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      70 CONTINUE                                                       DBMIE145
         WM1 = WFN(1)                                                   DBMIE146
         WFN(1) = WFN(2)                                                DBMIE147
         WFN(2) = (E(1)*AX*WFN(3) - WRL                                 DBMIE148
         TC1 = ALAPIN(1)*AXF + WMAX                                     DBMIE149
         TC2 = ALAPIN(M) + WMAX                                         DBMIE150
         FNA = (TC1*TA(3,3)-TA(2,3))/(TC1*WFN(2) - WFN(3))              DBMIE151
         FNB = (TC2*TA(2,3)-TA(1,3))/(TC2*WFN(2) - WFN(3))              DBMIE152
         T(3) = 0                                                       DBMIE153
         T(4) = T(1)/(E(3)*T(2))                                        DBMIE154
         T(2) = (T(2)*E(3) + E.GUU(1)/T(5)                              DBMIE155
         CTWO3 = CTWO3 + T(2)*T(U(1) + SU(2)*TBU(2) + TC(1)*TC(2) +     DBMIE156
        1 TC(2)*TC(2) + TU(2)*TBU(2) + TU(2)*TBU(2))                    DBMIE157
         WRT = WEXT + T(3)*(T(6)(1)+TC(1))                              DBMIE158
         T(4) = TU(1)**2 + TBU,2)**2 + TC(1)**2 + TC(2)**2              DBMIE159
         WSCAT = WSCAT + T(3) *T(4)                                     DBMIE160
         T(2) = W*W*W                                                   DBMIE161
         T(1) = T(1)/T(2)                                               DBMIE162
         K = K*K/2**2                                                   DBMIE163
         DO 83 J = 1,4                                                  DBMIE164
         ELTHMX(1,J,1)=ELTHMX(1,J,1)+T(1)*(TB(1)*P(1),J)+TC(1)*TAU(3,J))DBMIE165
         ELTHMX(2,J,1)=ELTHMX(2,J,1)+T(1)*(TB(2)*P(1),J)+TC(2)*TAU(3,J))DBMIE166
         ELTHMX(3,J,1)=ELTHMX(3,J,1)+T(1)*(TC(1)*P(1),J)+TB(1)*TAU(3,J))DBMIE167
         ELTHMX(4,J,1)=ELTHMX(4,J,1)+T(1)*(TC(2)*P(1),J)+TB(2)*TAU(3,J))DBMIE168
         IF ( M .EQ. N ) GO TO 85                                       DBMIE169
         ELTHMX(1,J,2)=ELTHMX(1,J,2)+T(1)*(TB(1)*P(1),J)-TC(1)*TAU(3,J))DBMIE170
         ELTHMX(2,J,2)=ELTHMX(2,J,2)+T(1)*(TB(2)*P(1),J)-TC(2)*TAU(3,J))DBMIE171
         ELTHMX(3,J,2)=ELTHMX(3,J,2)+T(1)*(TC(1)*P(1),J)-TB(1)*TAU(3,J))DBMIE172
         ELTHMX(4,J,2)=ELTHMX(4,J,2)+T(1)*(TC(2)*P(1),J)-TB(2)*TAU(3,J))DBMIE173
         GO TO 83                                                       DBMIE174
      85 ELTHMX(1,J,2)=ELTHMX(1,J,2)+T(1)*(-TB(1)*P(3,J)+TC(1)*TAU(3,J))DBMIE175
         ELTHMX(2,J,2)=ELTHMX(2,J,2)+T(1)*(-TB(2)*P(3,J)+TC(2)*TAU(3,J))DBMIE176
         ELTHMX(3,J,2)=ELTHMX(3,J,2)+T(1)*(-TC(1)*P(3,J)+TB(1)*TAU(3,J))DBMIE177
         ELTHMX(4,J,2)=ELTHMX(4,J,2)+T(1)*(-TC(2)*P(3,J)+TB(2)*TAU(3,J))DBMIE178
      80 CONTINUE                                                       DBMIE179
         IF( T(4) .LT. 1.0D-14 ) GO TO 100                              DBMIE180
         N = N + 1                                                      DBMIE181
         DO 90 J = 1,4                                                  DBMIE182
         P(1),J) = P(2,J)                                               DBMIE183
         P(2,J) = P(3,J)                                                DBMIE184
         TAU(1,J) = TAU(2,J)                                            DBMIE185
         TAU(2,J) = TAU(3,J)                                            DBMIE186
      90 CONTINUE                                                       DBMIE187
         WNAP = WNA                                                     DBMIE188
         WNBP = WNB                                                     DBMIE189
         IF ( N .LE. NMX2 ) GO TO 65                                    DBMIE190
         WRITE(6,8 )                                                    DBMIE191
         CALL EXIT                                                      DBMIE192
```



```
     100 DO 120 J = 1, 4
         DO 120 K = 1,2
         DO 115 I = 1,4
         T(I) = ELTHMX(I,J,K)
     115 CONTINUE
         ELTHMX(2,J,K) = E(1)**2 * T(2)**2
         ELTHMX(1,J,K) = E(3)**2 * E(4)**2
         ELTHMX(3,J,K) = E(3)*E(3) + E(2)*T(4)
         ELTHMX(4,J,K) = E(2)*E(3) - E(4)*T(1)
     120 CONTINUE
         T(1) = 2.0D0 * RA**2
         WEXT = WEXT * T(1)
         WSCAT = WSCAT * T(1)
         CTWO3 = 2.0D0 * CTWO3 * T(1)
C        THE DETAILS ABOUT THIS SUBROUTINE CAN BE FOUND IN THE FOLLOWING
C        REPORT : SUBROUTINES FOR COMPUTING THE PARAMETERS OF THE
C        ELECTROMAGNETIC RADIATION SCATTERED BY A SPHERE : J.V. DAVE,
C        I B M SCIENTIFIC CENTER, PALO ALTO , CALIFORNIA.
C        REPORT NO. 320 - 3237 , MAY 1968 .
         RETURN
         END
```

APPENDIX IV

Sample Output Using DAMIE Subroutine

m = 1.342 - 0.0i, r = 100.0 μ, λ = 0.4 μ

ELEMENTS OF THE TRANSFORMATION MATRIX FOR A SPHERE WITH SIZE PARAMETER = 1570.7963

REFRACTIVE INDEX, REAL = 0.134200 01 IMAGINARY 0.0

ANGLE	S SUB 2	S SUB 1	S SUB 21	D SUB 21	INTENSITY	POLARIZATION
0.	0.1542040 13	0.1542040 13	0.1542040 13	-0.0	0.1542040 13	0.0
1.0000	0.1627600 09	0.1612140 09	0.1627600 09	-0.1094270 07	0.1627600 09	0.0000
2.0000	0.4613880 08	0.4555040 08	0.4613880 08	-0.1243870 07	0.4613920 08	-0.0127
3.0000	0.7285940 07	0.6879420 07	0.7285940 07	-0.6769350 06	0.7337210 07	-0.0561
4.0000	0.2234730 07	0.1972010 07	0.2234730 07	-0.3031260 06	0.2274670 07	-0.1551
5.0000	0.3679420 07	0.3173050 07	0.3679420 07	-0.1703790 06	0.3684360 07	-0.0429
6.0000	0.1184140 08	0.1155960 08	0.1184140 08	-0.4192270 06	0.1184910 08	-0.0071
7.0000	0.1208690 08	0.1346950 08	0.1208690 08	-0.4164150 06	1.1208820 08	0.0991
8.0000	0.9427080 07	0.9462660 07	0.9427080 07	-0.3130760 06	0.9441650 07	-0.0338
9.0000	0.1134120 08	0.1134120 08	0.1134120 08	-0.2970040 06	0.1136590 08	-0.0822
10.0000	0.1061320 08	0.1154950 08	0.1061320 08	-0.1159030 07	0.1061490 08	-0.0849
11.0000	0.6797940 07	0.6959540 07	0.6797940 07	-0.4663450 06	0.6797944 07	-0.0454
12.0000	0.5493030 07	0.5992540 07	0.5493030 07	-0.3292820 06	0.5462200 07	-0.0052
13.0000	0.7487570 07	0.7526010 07	0.7487570 07	-0.1615140 06	0.7483420 07	0.0052
14.0000	0.9748240 07	0.9736040 07	0.9748240 07	-0.2624830 06	0.9751840 07	-0.0044
15.0000	0.6806990 07	0.6833480 07	0.6806990 07	-0.4764450 06	0.6806050 07	0.0112
16.0000	0.5640150 07	0.5552360 07	0.5640150 07	-0.3517920 06	0.5652570 07	-0.0229
17.0000	0.6224660 07	0.6815190 07	0.6224660 07	-0.2830700 06	0.6234660 07	0.0124
18.0000	0.7310610 07	0.7364670 07	0.7310610 07	-0.5171810 06	0.7331490 07	0.0246
19.0000	0.3753890 07	0.3741350 07	0.3753890 07	-0.2644950 06	0.3737180 07	-0.0068
20.0000	0.3537050 07	0.3648960 07	0.3537050 07	-0.3140780 06	0.3617470 07	-0.1295
21.0000	0.4266900 07	0.4443260 07	0.4266900 07	-0.3020860 06	0.4291750 07	-0.0812
22.0000	0.4573120 07	0.4482990 07	0.4573120 07	-0.3834220 06	0.4590470 07	-0.0238
23.0000	0.3325350 07	0.2972100 07	0.3325350 07	-0.1668480 06	0.3351160 07	-0.1135
24.0000	0.2342600 07	0.2019780 07	0.2342600 07	-0.2833570 05	0.2365270 07	-0.1376
25.0000	0.3386010 07	0.3076150 07	0.3386010 07	-0.2157380 06	0.3409170 07	-0.0977
26.0000	0.5286370 07	0.5674520 07	0.5286370 07	-0.9679510 05	0.5300040 07	0.0706
27.0000	0.2990940 07	0.2924770 07	0.2990940 07	-0.2991540 06	0.3006470 07	-0.0232
28.0000	0.2931570 07	0.2775330 07	0.2931570 07	-0.3954590 06	0.2984120 07	-0.0636
29.0000	0.3655680 07	0.3563980 07	0.3655680 07	-0.4819010 06	0.3689440 07	-0.0340
30.0000	0.1884040 07	0.1599460 07	0.1884040 07	-0.2090680 06	0.1933170 07	-0.1744
31.0000	0.2937470 07	0.2681950 07	0.2937470 07	-0.2631940 06	0.2949560 07	-0.0873
32.0000	0.3317670 07	0.3281550 07	0.3317670 07	-0.1705860 06	0.3322300 07	-0.0621
33.0000	0.1325900 07	0.1484860 06	0.1325900 07	-0.1170530 06	0.1400540 07	-0.3111
34.0000	0.2430060 07	0.2398610 07	0.2430060 07	-0.3868370 06	0.2461800 07	-0.0259
35.0000	0.2623750 07	0.2696490 07	0.2623750 07	-0.2998900 06	0.2641370 07	0.0202
36.0000	0.1054420 07	0.8155100 06	0.1054420 07	-0.3047470 06	0.1146560 07	-0.0892
37.0000	0.7030860 06	0.4760300 06	0.7030860 06	-0.2330750 06	0.8142960 06	-0.4154
38.0000	0.1341580 07	0.1427730 07	0.1341580 07	-0.4901370 06	0.1437180 07	-0.1109
39.0000	0.1867820 07	0.1892150 07	0.1867820 07	-0.3255450 06	0.1896230 07	0.0183
40.0000	0.1906670 07	0.2000440 07	0.1906670 07	-0.1578340 06	0.1915090 07	0.0446
41.0000	0.1887750 07	0.1981140 07	0.1887750 07	-0.5781520 05	0.1888640 07	-0.0038
42.0000	0.1594670 07	0.1569110 07	0.1594670 07	-0.1958450 06	0.1606740 07	-0.0110
43.0000	0.1365800 07	0.1415.30 07	0.1365800 07	-0.2930470 06	0.1399390 07	-0.0594
44.0000	0.1079760 07	0.1157[30 07	0.1079760 07	-0.2316890 06	0.1105350 07	-0.0429
45.0000	0.1114290 07	0.1094370 07	0.1114290 07	-0.2401520 06	0.1144290 07	-0.0838
46.0000	0.8651950 06	0.7930730 06	0.8651950 06	-0.2822380 06	0.9186960 06	-0.1367
47.0000	0.7292860 06	0.6359150 06	0.7292860 06	-0.1559960 06	0.7554930 06	-0.1561
48.0000	0.9271370 06	0.8746330 06	0.9271370 06	-0.1943670 06	0.9505210 06	-0.0746

	0.9425481D 06	0.1000148D 07	0.9085820D 06	-0.3140811D 06	0.9631995E 06	
	0.8738610D 06	0.9321840D 06	0.8433670D 06	-0.2319780D 06	0.7279788E 06	
	0.7168496D 06	0.8175830D 06	0.6543780D 06	-0.1708010D 06	0.6677277E 06	
	0.6599398D 06	0.6444980D 06	0.6308150D 06	-0.1358610D 06	0.6497611E 06	
	0.5780450D 06	0.5329110D 06	0.5985430D 06	-0.8044760D 05	0.6027888E 06	
	0.6696780D 06	0.6597960D 06	0.6772960D 06	-0.2578030D 05	0.6780406E 06	
	0.5938650D 06	0.6414970D 06	0.6126280D 06	0.4599160D 05	0.6195908E 06	
	0.5826110D 06	0.6241640D 06	0.6054420D 06	-0.2240450D 04	0.6058922E 06	
	0.5616530D 06	0.6044180D 06	0.5972580D 06	0.7133460D 05	0.6029186E 06	
	0.5568470D 06	0.5900480D 06	0.5499980D 06	0.6977220D 05	0.5922265E 06	
	0.5437140D 06	0.5647450D 06	0.5429540D 06	0.1308400D 06	0.5442664E 06	
	0.5343730D 06	0.5430840D 06	0.5013250D 06	0.1306420D 06	0.5066879E 06	
	0.5279050D 06	0.5299310D 06	0.5038200D 06	0.1116020D 06	0.5063562E 06	
	0.5206530D 06	0.5278660D 06	0.5077240D 06	0.1051160D 06	0.5092611E 06	
	0.5248150D 06	0.5248770D 06	0.5227852D 06	0.8062580D 05	0.5217797E 06	
	0.5229650D 06	0.5226552D 06	0.5209899D 06	0.8402590D 05	0.5226093E 06	
	0.5248510D 06	0.5243528D 06	0.5226920D 06	0.8521590D 05	0.5242195E 06	
	0.5298640D 06	0.5251720D 06	0.5212963D 06	0.9577510D 05	0.5279726E 06	
	0.5359140D 06	0.5320530D 06	0.5278760D 06	0.9498128D 05	0.5342599E 06	
	0.5550130D 06	0.5491470D 06	0.5478720D 06	0.1078726D 06	0.5484159E 06	
	0.5748040D 06	0.5720520D 06	0.5697880D 06	0.1152068D 06	0.5709785E 06	
	0.5966150D 06	0.5943390D 06	0.5926480D 06	0.1320688D 06	0.5935211E 06	
	0.6157480D 06	0.6120840D 06	0.6102960D 06	0.1520688D 06	0.6119858E 06	
	0.6360370D 05	0.6673070D 05	0.6862460D 05	-0.5018790D 05	0.7519722E 05	
	0.5899230D 05	0.5988940D 05	0.7197830D 04	-0.4787430D 05	0.5041596E 05	
	0.4198150D 05	0.4943390D 05	-0.2089830D 05	-0.1109780D 06	0.2619885E 05	
	0.3966740D 05	0.3904780D 05	0.5756740D 04	0.3034550D 05	0.3688047E 05	
	0.2867160D 05	0.2782710D 05	0.3978210D 05	0.6535840D 05	0.5314992E 05	
	0.2716140D 05	0.2629650D 05	0.4671330D 05	-0.7982950D 04	0.5494395E 05	
	0.2221780D 05	0.2196740D 05	0.1723240D 05	-0.3108200D 05	0.3956946E 05	
	0.1726860D 05	0.1700120D 05	-0.1741190D 05	-0.6552440D 04	0.1865545E 05	
	0.1492180D 05	0.1431980D 05	0.2635750D 04	0.2167450D 05	0.2540006E 05	
	0.8892490D 04	0.9448570D 05	0.2187990D 05	-0.2445050D 04	0.3182911E 05	
	0.1254780D 05	0.2305260D 05	-0.5514830D 04	-0.1608150D 05	0.1779522E 05	
	0.1070410D 05	0.2220920D 05	-0.8926170D 04	0.1458330D 05	0.1900414E 05	
	0.2166850D 04	0.6321070D 05	0.1164020D 05	0.1240110D 04	0.3206098E 05	
	0.7859910D 04	0.4106880D 05	-0.1143330D 05	-0.1385350D 05	0.2445543E 05	
	0.6597350D 04	0.3656790D 05	-0.7476490D 04	0.1311580D 05	0.2057246E 05	
	0.1149860D 04	0.5150580D 05	0.3813060D 04	-0.6683840D 04	0.2632778E 05	
	0.6023810D 04	0.2415730D 05	-0.1205280D 05	-0.4997860D 03	0.1509066E 05	
	0.4546660D 02	0.4578290D 05	-0.1669640D 04	-0.9842230D 03	0.2291426E 05	
	0.6757990D 04	0.2476810D 05	-0.1270460D 05	0.2434590D 04	0.1575995E 05	
	0.3270850D 02	0.4818880D 05	0.3590720D 02	-0.1186280D 04	0.2161588E 05	
	0.5816960D 04	0.4229720D 05	-0.1554970D 05	0.2040860D 04	0.2405716E 05	
	0.3102540D 04	0.2187790D 05	-0.4698130D 04	0.1411800D 04	0.1146508E 05	
	0.2869920D 04	0.2361520D 05	-0.8203320D 04	-0.3713520D 02	0.1323186E 05	
	0.5433460D 04	0.1070310D 05	-0.1113520D 05	-0.6594480D 04	0.1806884E 05	
	0.5703360D 03	0.5380080D 05	-0.4120730D 04	0.1517280D 04	0.1718922E 05	
	0.7104840D 04	0.3666260D 05	-0.1546540D 05	-0.4458820D 04	0.2178376E 05	
	0.1931070D 04	0.3071490D 05	-0.4999790D 04	-0.3211620D 04	0.1652268E 05	
	0.4083840D 04	0.2366750D 05	-0.8926350D 04	-0.4106490D 04	0.1386921E 05	
	0.2093090D 04	0.1644190D 05	-0.5239930D 04	0.2837700D 04	0.9267478E 04	
	0.4750670D 04	0.2849480D 05	-0.1502950D 05	0.7181040D 04	0.1910231E 05	
	0.3976340D 04	0.3961720D 05	-0.1057320D 05	-0.6703980D 04	0.2169678E 05	
	0.3292420D 04	0.1924790D 05	-0.7960440D 04	-0.3679020D 03	0.1129028E 05	
	0.7338330D 04	0.3112500D 05	-0.1503770D 05	0.1487310D 04	0.1923307E 05	
	0.1853160D 04	0.3095430D 05	-0.4252070D 04	-0.6267810D 04	0.1640337E 05	
	0.3099920D 04	0.3683700D 05	-0.5436690D 04	0.9199690D 04	0.1996485E 05	
	0.1182860D 05	0.1781860D 05	-0.1449900D 05	0.7405220D 03	0.1482306E 05	
	0.1181000D 05	0.4656070D 05	-0.2163430D 05	-0.8428360D 04	0.2873036E 05	
	0.4435870D 04	0.2063550D 05	-0.1022800D 05	-0.5581150D 04	0.1752078E 05	
	0.3219630D 03	0.3339420D 05	-0.2219200D 04	-0.8175170D 03	0.7830584E 04	

180.0000	0.1447160D 04	0.1476640D 05	-0.5478970D 04	0.4564360D 04	0.9106814E 04	
181.0000	0.9320870D 04	0.1491270D 05	-0.9864260D 04	0.6031640D 04	0.1162041E 05	
182.0000	0.1372840D 05	0.1197090D 05	-0.1122520D 05	0.6099580D 04	0.1284941E 05	
183.0000	0.1904790D 05	0.1172150D 05	-0.1485120D 05	0.1644980D 04	0.1538445E 05	
184.0000	0.2772460D 05	0.1487060D 04	-0.9997140D 04	-0.2755410D 04	0.1560176E 05	
185.0000	0.3118970D 05	0.3368190D 04	0.4868750D 04	0.4884680D 04	0.1726899E 05	
186.0000	0.2892970D 05	0.3313270D 04	-0.2890130D 04	0.2640000D 05	0.2753122E 05	
187.0000	0.1959750D 05	0.3406930D 05	-0.2182070D 05	0.1363490D 05	0.2683335E 05	
188.0000	0.2151070D 05	0.1162020D 05	0.8343830D 04	0.1362900D 05	0.1856594E 05	
189.0000	0.5828380D 04	0.1268440D 04	0.1513810D 05	0.2149180D 05	0.6613604E 05	
190.0000	0.3647000D 03	0.2734540D 05	0.1940550D 04	-0.3433170D 04	0.1397686E 05	
191.0000	0.8895600D 04	0.1401270D 05	0.1206420D 05	-0.3181210D 05	0.6991111E 05	
192.0000	0.5123070D 05	0.6703280D 03	0.3505880D 04	0.3437150D 04	0.2585066E 05	
193.0000	0.5382520D 05	0.2474930D 05	0.3995440D 05	-0.2647590D 04	0.4180892E 05	
194.0000	0.1711100D 05	0.4663990D 04	0.2730650D 04	-0.2730490D 04	0.3186902E 05	
195.0000	0.1628210D 05	0.2812840D 04	0.6540540D 05	-0.1537390D 05	0.1468783E 06	
196.0000	0.4501840D 05	0.2014450D 06	0.3833520D 05	-0.8967210D 05	0.1234615E 06	
197.0000	0.3035490D 05	0.3344140D 05	0.2100270D 05	-0.2534400D 05	0.5300838E 05	
198.0000	0.9994470D 04	0.1444300D 05	-0.2678810D 04	0.1171260D 05	0.1221908E 05	
199.0000	0.4840280D 04	0.1617880D 05	-0.9032080D 04	0.8096760D 04	0.1760911E 05	
200.0000	0.2126270D 05	0.4016470D 04	-0.1292760D 05	-0.4034240D 04	0.1499488E 05	
201.0000	0.3572490D 04	0.1400410D 05	-0.9485360D 04	-0.2784840D 04	0.1144038E 05	
202.0000	0.1801620D 05	0.2249410D 05	-0.1765120D 05	-0.9821850D 04	0.2033521E 05	
203.0000	0.8408370D 04	0.4497730D 05	-0.1370050D 05	-0.1448650D 05	0.2819721E 05	
204.0000	0.4708330D 04	0.4688640D 05	-0.1489420D 05	0.1284680D 05	0.2578766E 05	
205.0000	0.1197000D 05	0.4741380D 05	-0.2434090D 05	-0.8374050D 04	0.3069306E 05	
206.0000	0.1206640D 05	0.5132680D 05	-0.1145480D 05	-0.2209180D 05	0.3169866E 05	
207.0000	0.5013940D 04	0.5403760D 05	-0.7724610D 04	-0.1694240D 05	0.2956678E 05	
208.0000	0.2454650D 04	0.4967700D 05	-0.1006320D 05	0.4546870D 04	0.2606588E 05	
209.0000	0.8947650D 05	0.5143810D 06	0.2053690D 06	-0.6218230D 05	0.3020295E 06	
210.0000	0.1268250D 06	0.2249050D 07	0.4735270D 06	-0.2590290D 06	0.1211946E 07	
211.0000	0.9556160D 05	0.5932710D 05	0.6656080D 05	-0.3520000D 05	0.7744462E 05	
212.0000	0.1690190D 06	0.8409800D 06	0.1835650D 05	0.2211930D 05	0.8771446E 05	
213.0000	0.1657620D 06	0.2127880D 06	0.3028240D 05	0.1878400D 06	0.1918506E 06	
214.0000	0.2841470D 06	0.9485240D 05	-0.1915500D 05	-0.1630480D 06	0.1894996E 06	
215.0000	0.6012400D 05	0.4612260D 06	0.1625140D 06	-0.1631170D 05	0.2606756E 06	
216.0000	0.9092650D 05	0.2412130D 06	0.1128840D 06	0.9588410D 05	0.1660697E 06	
217.0000	0.7761280D 05	0.2089450D 06	0.8252140D 05	-0.9884980D 05	0.1431046E 06	
218.0000	0.2522200D 06	0.2469410D 05	0.5213610D 05	0.5917220D 05	0.1384402E 06	
219.0000	0.4327240D 05	0.5707380D 06	0.3693150D 05	0.1527530D 06	0.1070055E 06	
220.0000	0.9840530D 05	0.2925310D 06	0.1576360D 06	-0.9921210D 05	0.1955468E 06	
221.0000	0.3136580D 05	0.4986810D 06	0.6955760D 05	-0.1145250D 06	0.2639138E 06	
222.0000	0.4052800D 05	0.2529890D 06	0.1005340D 06	0.1198690D 05	0.1467238E 06	
223.0000	0.2799660D 05	0.2671010D 06	0.8403720D 05	-0.2038460D 05	0.1475498E 06	
224.0000	0.1007350D 06	0.1496870D 06	0.1346100D 05	0.1401810D 06	0.1488044E 06	
225.0000	0.2119860D 06	0.3828890D 06	-0.1387620D 05	-0.2468010D 05	0.1079027E 06	
226.0000	0.1696410D 05	0.1540010D 06	0.4465120D 05	-0.2687490D 05	0.8548277E 05	
227.0000	0.1949310D 06	0.7902870D 04	-0.2300410D 05	-0.3180140D 05	0.1014176E 06	
228.0000	0.8329060D 04	0.2889210D 06	-0.3400810D 05	0.3628930D 05	0.1466256E 06	
229.0000	0.3717280D 05	0.5736310D 05	0.1156590D 05	-0.4471370D 05	0.4727798E 05	
230.0000	0.1798690D 06	0.4582970D 05	-0.6889640D 05	0.5912640D 05	0.1128396E 06	
231.0000	0.1886860D 06	0.7879870D 05	-0.7626590D 05	0.9614280D 05	0.1362426E 06	
232.0000	0.1347540D 06	0.7902390D 05	-0.6542610D 05	0.7980090D 05	0.1068896E 06	
233.0000	0.1102450D 06	0.8873820D 05	-0.6594000D 05	0.7369900D 05	0.9967656E 05	
234.0000	0.1485810D 06	0.3716790D 05	-0.7529720D 05	0.7615100D 05	0.1128846E 06	
235.0000	0.2269330D 06	0.7729120D 05	-0.1319010D 06	0.1191710D 06	0.1521121E 06	
236.0000	0.1009750D 06	0.2832990D 05	-0.4781340D 05	-0.2396850D 05	0.6465246E 05	
237.0000	0.1308880D 05	0.1279940D 06	-0.5780180D 05	0.3028680D 05	0.8054136E 05	
238.0000	0.7930760D 05	0.5269360D 05	-0.3716330D 05	0.3481020D 05	0.5600086E 05	
239.0000	0.6532470D 05	0.1502730D 06	-0.9685850D 05	-0.2085620D 05	0.1077996E 06	
240.0000	0.1490840D 06	0.7702980D 05	-0.1071510D 06	0.1627380D 06	0.1130576E 06	

[illegible]

EFFICIENCY FACTOR FOR EXTINCTION 0.2012960 01

EFFICIENCY FACTOR FOR SCATTERING 0.2012960 01

EFFICIENCY FACTOR FOR ABSORPTION 0.0

ASYMMETRY FACTOR [illegible]

TOTAL TIME FOR THIS CASE IN SECONDS = 299.984

APPENDIX V

Sample Output Using DBMIE Subroutine

m = 1.342 - 0.1i, r = 100.0 μ, λ = 0.4 μ

ELEMENTS OF THE TRANSFORMATION MATRIX FOR A SPHERE WITH SIZE PARAMETER = 1570.7963

REFRACTIVE INDEX, REAL = 0.134200 01 IMAGINARY 0.100000 00

ANGLE	M SUB 2	M SUB 1	S SUB 21	D SUB 21	INTENSITY	POLARIZATION
0.0	0.1544270 13	0.1544270 13	0.1544270 13	-0.0	0.1544271E 13	0.0
1.0000	0.1914000 09	0.1913720 09	0.1914880D 09	0.2102360 06	0.1914891E 09	[illegible]
2.0000	0.1571111 08	0.1561250 08	0.1488080 08	-0.3766990 06	0.1466066E 08	[illegible]
3.0000	0.1251130 07	0.1560570 07	0.1425080 07	-0.4555170 05	0.1433395E 07	[illegible]
4.0000	0.1991170 07	0.2262060 07	0.2121590 07	0.4460710 05	0.2126901E 07	[illegible]
5.0000	0.2005610 07	0.2223380 07	0.2110240 07	-0.3665000 04	0.2112297E 07	[illegible]
6.0000	0.1104770 07	0.1220120 07	0.1132810 07	-0.4475380 05	0.1137722E 07	[illegible]
7.0000	0.5590630 06	0.7052900 06	0.6127380 06	-0.1777070 05	0.6194621E 06	[illegible]
8.0000	0.5505760 06	0.7294020 06	0.6241960 06	0.2909340 04	0.6348171E 06	[illegible]
9.0000	0.5578540 06	0.7067010 06	0.6322000 06	-0.9090530 04	0.6372061E 06	[illegible]
10.0000	0.4522190 06	0.5271120 06	0.4917640 06	-0.1906210 05	0.4979301E 06	[illegible]
11.0000	0.3270640 06	0.4784420 06	0.3957800 06	-0.3169150 05	0.4030041E 06	0.181
12.0000	0.3435490 06	0.4491420 06	0.3909160 06	-0.6082600 04	0.3938421E 06	0.191
13.0000	0.3192270 06	0.4494030 06	0.3076510 06	-0.1140400 05	0.3752308E 06	0.197
14.0000	0.2564280 06	0.4031510 06	0.3213290 06	-0.1324200 05	0.3298881E 06	0.22
15.0000	0.2287940 06	0.1761410 06	0.2931840 06	-0.1030590 05	0.3024508E 06	0.241
16.0000	0.2178730 06	0.3650470 06	0.2868570 06	-0.1000360 05	0.2914882E 06	0.25
17.0000	0.1997210 06	0.3461620 06	0.2626750 06	-0.1173120 05	0.2729421E 06	0.26
18.0000	0.1795480 06	0.3257960 06	0.2415810 06	-0.1127770 05	0.2526511E 06	0.28
19.0000	0.1672360 06	0.3113590 06	0.2260860 06	-0.1045900 05	0.2402298E 06	0.30
20.0000	0.1596000 06	0.3013340 06	0.2143990 06	-0.1102220 05	0.2265711E 06	0.31
21.0000	0.1425130 06	0.2877910 06	0.2020720 06	-0.1111340 05	0.2149952E 06	0.33
22.0000	0.1319310 06	0.2762130 06	0.1906000 06	-0.1063970 05	0.2042038E 06	0.35
23.0000	0.1227590 06	0.2662070 06	0.1808520 06	-0.1074970 05	0.1944808E 06	0.36
24.0000	0.1131600 06	0.2555340 06	0.1696700 06	-0.1075990 05	0.1843241E 06	0.38
25.0000	0.1049900 06	0.2463780 06	0.1601630 06	-0.1048040 05	0.1753384E 06	0.40
26.0000	0.9780270 05	0.2372230 06	0.1514230 06	-0.1045500 05	0.1671801E 06	0.41
27.0000	0.8961830 05	0.2264220 06	0.1427010 06	-0.1034810 05	0.1580191E 06	0.43
28.0000	0.8292510 05	0.2203010 06	0.1347800 06	-0.1014770 05	0.1516841E 06	0.45
29.0000	0.7662340 05	0.2124920 06	0.1272450 06	-0.1005670 05	0.1445631E 06	0.46
30.0000	0.7074630 05	0.2054210 06	0.1200010 06	-0.9881540 04	0.1378371E 06	0.48
31.0000	0.6534680 05	0.1973130 06	0.1132820 06	-0.9717070 04	0.1315841E 06	0.50
32.0000	0.6024290 05	0.1904140 06	0.1068160 06	-0.9564110 04	0.1255781E 06	0.52
33.0000	0.5551770 05	0.1843390 06	0.1007280 06	-0.9377110 04	0.1199281E 06	0.53
34.0000	0.5111490 05	0.1783270 06	0.9495080 05	-0.9252920 04	0.1145731E 06	0.55
35.0000	0.4700840 05	0.1719580 06	0.8945240 05	-0.9024520 04	0.1094821E 06	0.57
36.0000	0.4319740 05	0.1661530 06	0.8425560 05	-0.8844860 04	0.1046741E 06	0.58
37.0000	0.3963620 05	0.1605570 06	0.7930280 05	-0.8657170 04	0.1000961E 06	0.60
38.0000	0.3633540 05	0.1551981 06	0.7462540 05	-0.8469940 04	0.9576881E 05	0.62
39.0000	0.3325760 05	0.1500440 06	0.7015290 05	-0.8281100 04	0.9164921E 05	0.63
40.0000	0.3040330 05	0.1450930 06	0.6592240 05	-0.8091610 04	0.8774811E 05	0.65
41.0000	0.2774800 05	0.1403380 06	0.6189920 05	-0.7902130 04	0.8404031E 05	0.66
42.0000	0.2528640 05	0.1357610 06	0.5808120 05	-0.7713140 04	0.8052371E 05	0.68
43.0000	0.2300110 05	0.1313640 06	0.5445100 05	-0.7524520 04	0.7718261E 05	0.70
44.0000	0.2088470 05	0.1271330 06	0.5100400 05	-0.7337650 04	0.7401181E 05	0.71
45.0000	0.1892410 05	0.1230740 06	0.4772770 05	-0.7151500 04	0.7099921E 05	0.73
46.0000	0.1711090 05	0.1191660 06	0.4461500 05	-0.6967820 04	0.6813861E 05	0.74
47.0000	0.1543540 05	0.1154080 06	0.4165730 05	-0.6785580 04	0.6542171E 05	0.76
48.0000	0.1388480 05	0.1117920 06	0.3884620 05	-0.6605880 04	0.6284061E 05	0.77

49.0000	0.1246340 05	0.1083130 06	0.3617510 05	-0.6428570 04	0.6038901E 05
50.0000	0.1115110 05	0.1049680 06	0.3363630 05	-0.6251750 04	0.5805971E 05
51.0000	0.9945370 04	0.1017480 06	0.3122350 05	-0.6081820 04	0.5584671E 05
52.0000	0.8838540 04	0.9866940 05	0.2893040 05	-0.5912730 04	0.5374421E 05
53.0000	0.7825270 04	0.9566770 05	0.2675070 05	-0.5746600 04	0.5174651E 05
54.0000	0.6899270 04	0.9276830 05	0.2467900 05	-0.5583600 04	0.4984811E 05
55.0000	0.6055040 04	0.9003350 05	0.2270990 05	-0.5423750 04	0.4804421E 05
56.0000	0.5287300 04	0.8737280 05	0.2083820 05	-0.5267100 04	0.4633011E 05
57.0000	0.4591130 04	0.8481070 05	0.1905900 05	-0.5113720 04	0.4470101E 05
58.0000	0.3962330 04	0.8234380 05	0.1736770 05	-0.4963650 04	0.4315281E 05
59.0000	0.3396200 04	0.7996000 05	0.1576010 05	-0.4816890 04	0.4168151E 05
60.0000	0.2889120 04	0.7767730 05	0.1423190 05	-0.4673450 04	0.4028311E 05
61.0000	0.2436180 04	0.7547620 05	0.1277920 05	-0.4533330 04	0.3895411E 05
62.0000	0.2034880 04	0.7336030 05	0.1139830 05	-0.4396540 04	0.3769091E 05
63.0000	0.1681590 04	0.7129910 05	0.1008560 05	-0.4263080 04	0.3649031E 05
64.0000	0.1373040 04	0.6932940 05	0.8837770 04	-0.4132860 04	0.3534921E 05
65.0000	0.1106380 04	0.6742240 05	0.7651580 04	-0.4005920 04	0.3426461E 05
66.0000	0.8787190 03	0.6553080 05	0.6524600 04	-0.3882210 04	0.3323371E 05
67.0000	0.6874950 03	0.6382030 05	0.5452260 04	-0.3761610 04	0.3225401E 05
68.0000	0.5302260 03	0.6211920 05	0.4433310 04	-0.3644240 04	0.3132271E 05
69.0000	0.4065900 03	0.6047050 05	0.3466850 04	-0.3530020 04	0.3043761E 05
70.0000	0.3088290 03	0.5886440 05	0.2544300 04	-0.3418800 04	0.2959691E 05
71.0000	0.2396830 03	0.5733530 05	0.1669420 04	-0.3310590 04	0.2879881E 05
72.0000	0.1964200 03	0.5586770 05	0.8376740 03	-0.3205310 04	0.2803891E 05
73.0000	0.1768010 03	0.5444520 05	0.4725380 02	-0.3102910 04	0.2731841E 05
74.0000	0.1742480 03	0.5307780 05	-0.7033690 03	-0.3003410 04	0.2662841E 05
75.0000	0.2021110 03	0.5175340 05	-0.1417900 04	-0.2906710 04	0.2597621E 05
76.0000	0.2458[illegible]0 03	0.5048410 05	-0.2096170 04	-0.2812610 04	0.2535641E 05
77.0000	0.3030[illegible]0 03	0.4925200 05	-0.2741090 04	-0.2721310 04	0.2476751E 05
78.0000	0.3784[illegible]0 03	0.4807140 05	-0.3353720 04	-0.2632610 04	0.2420791E 05
79.0000	0.4687120 03	0.4694100 05	-0.3935800 04	-0.2546410 04	0.2367621E 05
80.0000	0.5727[illegible]0 03	0.4576[illegible]0 05	-0.4488840 04	-0.2462710 04	0.2317111E 05
81.0000	0.6894[illegible]0 03	0.4467120 05	-0.5014240 04	-0.2381510 04	0.2269121E 05
82.0000	0.8177[illegible]0 03	0.4365[illegible]0 05	-0.5513340 04	-0.2302610 04	0.2223941E 05
83.0000	0.9567760 03	0.4264[illegible]0 05	-0.5987420 04	-0.2226010 04	0.2180241E 05
84.0000	0.1105510 04	0.4167690 05	-0.6437690 04	-0.2151710 04	0.2139121E 05
85.0000	0.1263130 04	0.4073820 05	-0.6865330 04	-0.2079600 04	0.2100071E 05
86.0000	0.1428840 04	0.3983380 05	-0.7271360 04	-0.2009770 04	0.2062981E 05
87.0000	0.1601840 04	0.3895580 05	-0.7656910 04	-0.1941840 04	0.2027771E 05
88.0000	0.1781560 04	0.3813530 05	-0.8022930 04	-0.1875990 04	0.1994341E 05
89.0000	0.1967230 04	0.3728530 05	-0.8370180 04	-0.1812120 04	0.1962611E 05
90.0000	0.2158180 04	0.3649170 05	-0.8700150 04	-0.1750170 04	0.1932491E 05
91.0000	0.2353930 04	0.3572480 05	-0.9013110 04	-0.1690090 04	0.1903911E 05
92.0000	0.2553880 04	0.3498200 05	-0.9310050 04	-0.1631820 04	0.1876791E 05
93.0000	0.2757530 04	0.3426530 05	-0.9591750 04	-0.1575330 04	0.1851061E 05
94.0000	0.2964390 04	0.3356840 05	-0.9858960 04	-0.1520550 04	0.1826661E 05
95.0000	0.3174020 04	0.3289630 05	-0.1011240 05	-0.1467430 04	0.1803521E 05
96.0000	0.3385940 04	0.3224940 05	-0.1035270 05	-0.1415940 04	0.1781571E 05
97.0000	0.3599840 04	0.3161940 05	-0.1058050 05	-0.1366020 04	0.1760761E 05
98.0000	0.3815370 04	0.3100350 05	-0.1079640 05	-0.1317640 04	0.1741051E 05
99.0000	0.4032060 04	0.3041520 05	-0.1100090 05	-0.1270730 04	0.1722361E 05
100.0000	0.4249650 04	0.2984340 05	-0.1119480 05	-0.1225270 04	0.1704651E 05
101.0000	0.4467830 04	0.2928990 05	-0.1137840 05	-0.1181260 04	0.1687891E 05
102.0000	0.4686120 04	0.2875380 05	-0.1155220 05	-0.1138500 04	0.1672011E 05
103.0000	0.4904840 04	0.2823460 05	-0.1171680 05	-0.1097120 04	0.1656971E 05
104.0000	0.5123150 04	0.2773180 05	-0.1187250 05	-0.1057010 04	0.1642751E 05
105.0000	0.5341020 04	0.2724470 05	-0.1201990 05	-0.1018150 04	0.1629291E 05
106.0000	0.5558220 04	0.2677280 05	-0.1215930 05	-0.9804420 03	0.1616551E 05
107.0000	0.5774550 04	0.2631570 05	-0.1229110 05	-0.9440030 03	0.1604511E 05
108.0000	0.5989820 04	0.2587280 05	-0.1241560 05	-0.9086500 03	0.1593131E 05
109.0000	0.6203870 04	0.2544370 05	-0.1253330 05	-0.8743970 03	0.1582381E 05

		-0.1206650 05	-0.8922130 03	0.1572221 05	
		-0.1274950 05	-0.8079660 03	0.1562831 05	
		-0.1298860 05	-0.7777430 03	0.1553398 05	
		-0.1296210 05	-0.7477650 03	0.1543941 05	
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		-0.1314160 05	-0.6662850 03	0.1514224 05	
		-0.1326530 05	-0.6362750 03	0.1503951 05	
		-0.1333480 05	-0.6010990 03	0.1493971 05	
		-0.1339990 05	-0.5657080 03	0.1483264 05	
		-0.1346120 05	-0.5616120 03	0.1472601 05	
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		-0.1362770 05	-0.4998650 03	0.1442135 05	
		-0.1367130 05	-0.4726900 03	0.1432000 05	
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		-0.1375770 05	-0.4329070 03	0.1411050 05	
		-0.1379080 05	-0.4132410 03	0.1400430 05	
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		-0.1392690 05	-0.3426990 03	0.1359080 05	
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		-0.1400010 05	-0.2956340 03	0.1327860 05	
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		-0.1404880 05	-0.2668770 03	0.1307901 05	
		-0.1406780 05	-0.2532340 03	0.1297616 05	
		-0.1408540 05	-0.2400640 03	0.1287595 05	
		-0.1410160 05	-0.2273540 03	0.1277905 05	
		-0.1411660 05	-0.2150930 03	0.1268111 05	
		-0.1413040 05	-0.2032690 03	0.1258494 05	
		-0.1414300 05	-0.1918710 03	0.1248952 05	
		-0.1415460 05	-0.1808860 03	0.1239438 05	
		-0.1416520 05	-0.1703100 03	0.1229984 05	
		-0.1417490 05	-0.1601280 03	0.1220718 05	
		-0.1418370 05	-0.1503330 03	0.1211564 05	
		-0.1419180 05	-0.1409080 03	0.1202483 05	
		-0.1419910 05	-0.1318550 03	0.1193584 05	
		-0.1420570 05	-0.1231610 03	0.1184906 05	
		-0.1421180 05	-0.1148170 03	0.1176021 05	
		-0.1421690 05	-0.1068160 03	0.1167038 05	
		-0.1422130 05	-0.9915450 02	0.1158094 05	
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		-0.1424110 05	-0.5983140 02	0.1104325 05	
		-0.1424310 05	-0.5429870 02	0.1095194 05	
		-0.1424480 05	-0.4903610 02	0.1086408 05	
		-0.1424630 05	-0.4402180 02	0.1077485 05	
		-0.1424750 05	-0.3925810 02	0.1068731 05	
		-0.1424860 05	-0.3521900 02	0.1059841 05	
		-0.1424950 05	-0.3112060 02	0.1051332 05	
		-0.1425020 05	-0.2726860 02	0.1042498 05	
		-0.1425080 05	-0.2372010 02	0.1034348 05	
		-0.1425130 05	-0.2041120 02	0.1025896 05	
		-0.1425160 05	-0.1735920 02	0.1017536 05	
		-0.1425190 05	-0.1450130 02	0.1009338 05	
		-0.1425220 05	-0.1201500 02	0.1001331 05	

		-0.1425230 05	-0.9717960 01	0.1425291 05	
		-0.1425250 05	-0.7068410 01	0.1425281 05	
		-0.1425250 05	-0.5065240 01	0.1425271 05	
		-0.1425260 05	-0.3405220 01	0.1425274 05	
		-0.1425260 05	-0.2286980 01	0.1425271 05	
		-0.1425260 05	-0.1410190 01	0.1425271 05	
		-0.1425260 05	-0.6174720 01	0.1425201 05	
		-0.1425260 05	-0.4701490 00	0.1425201 05	
		-0.1425260 05	-0.1405920 00	0.1425201 05	
		-0.1425260 05	-0.0	0.1425264 05	

EFFICIENCY FACTOR FOR DIFFRACTION = 0.2014440 01

EFFICIENCY FACTOR FOR SCATTERING = 0.1080490 01

EFFICIENCY FACTOR FOR ABSORPTION = 0.9339450 00

ASYMMETRY FACTOR = 0.9065730 00

TOTAL TIME FOR THIS CASE IN SECONDS = 303.792

AUTHOR(S)

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TITLE

Subroutines for Computing the Parameters of the Electromagnetic Radiation Scattered by a Sphere

PERFORMING DEPARTMENT

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ABSTRACT

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Two experimental FORTRAN subroutines with which one can compute the so-called "Efficiency Factors" and the Stokes parameters of the electromagnetic radiation scattered by a sphere, are described in great detail. The index of refraction (real part) of the material of the sphere is assumed to have the form $n_p - ik_p$ with $k_p \geq 0$. The formulas used for these subroutines were first derived by G. Mie in 1908 and as such this scattering process is referred to as Mie scattering in the scientific literature.

A listing of these subroutines along with sample program outputs are provided in the appendixes.

REMARKS

Available from IBM Program Information Department, Hawthorne, N. Y. PID No. 5600-17.1.002.

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